Thus we have an eigenvector basis: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

3   2   2

So 3. $A = \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 3 & & \\ & 2 & \\ & & 2 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{P^{-1}}$

$[P \,|\, I] \rightsquigarrow [I \,|\, P^{-1}]$

What can go wrong?

Ex $A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \xrightarrow[\text{eigenvalues}]{\text{compute}}$ $(4-\lambda)(2-\lambda)+1 = 0$ $\det(A-\lambda I)$

$\Rightarrow \lambda^2 - 6\lambda + 9 = 0$

$\Rightarrow (\lambda-3)^2 = 0$ ← root 3 has multiplicity 2

Compute 3-eigenspace:

$A - 3I = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} \boxed{1} & 1 \\ 0 & 0 \end{bmatrix}$ free

$x_1 = -x_2$
$x_2 =$ free

$\rightsquigarrow$ basis of 3-eigenspace is $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$ 1-dim'l.

1

Issue: dimension of 3-eigenspace is less than the multiplicity of 3 as an eigenvalue.

Thm Let A be an $n \times n$ matrix with distinct eigenvalues $\lambda_1, \ldots, \lambda_r$. $(r \leq n)$

1. For each λ_i, the dimension of the λ_i-eigenspace is less than or equal to the multiplicity of λ_i as an eigenvalue.

2. A is diagonalizable if and only if the sum of the dimensions of distinct eigenspaces is n.

   says for each i, dimension of λ_i-eigenspace equals multiplicity of λ_i as an eigenvalue.

3. If A is diagonalizable, the union of the bases of the distinct eigenspaces forms an eigenvector basis of A for $\mathbb{R}^n$.

   $A = PDP^{-1}$