Now, we can make sense of $A = PDP^{-1}$.

matrix of A-eigenvectors

diagonal

$P = P_{\mathcal{B}}$

$$
\begin{tikzcd}
V \arrow[r, "T"] \arrow[d, "\text{coord}"'] & W \arrow[d, "\text{coord}"] \\
\mathbb{R}^n \arrow[r, dashed, "\text{mult by } {}_{\mathcal{F}}[T]_{\mathcal{B}}"'] & \mathbb{R}^p
\end{tikzcd}
$$

Consider:

$$
\begin{tikzcd}
\bar{x} \arrow[rrr, mapsto, bend left] \arrow[ddd, bend right, "1"'] & & & A\bar{x} = PDP^{-1}\bar{x} \\
& (\mathbb{R}^n, \mathcal{E}) \arrow[r, "\text{mult by } A"] \arrow[d, "\text{mult by } P^{-1}"'] & (\mathbb{R}^n, \mathcal{E}) & \\
& (\mathbb{R}^n, \mathcal{B}) \arrow[r, "\text{mult by } D"] & (\mathbb{R}^n, \mathcal{B}) \arrow[u, "\text{mult by } P"'] & \\
P^{-1}\bar{x} = [\bar{x}]_{\mathcal{B}} \arrow[rrr, mapsto, bend right, "2"'] & & & DP^{-1}\bar{x} = D[\bar{x}]_{\mathcal{B}} \arrow[uuu, bend right, "3"']
\end{tikzcd}
$$

easy and clear b/c D is diagonal

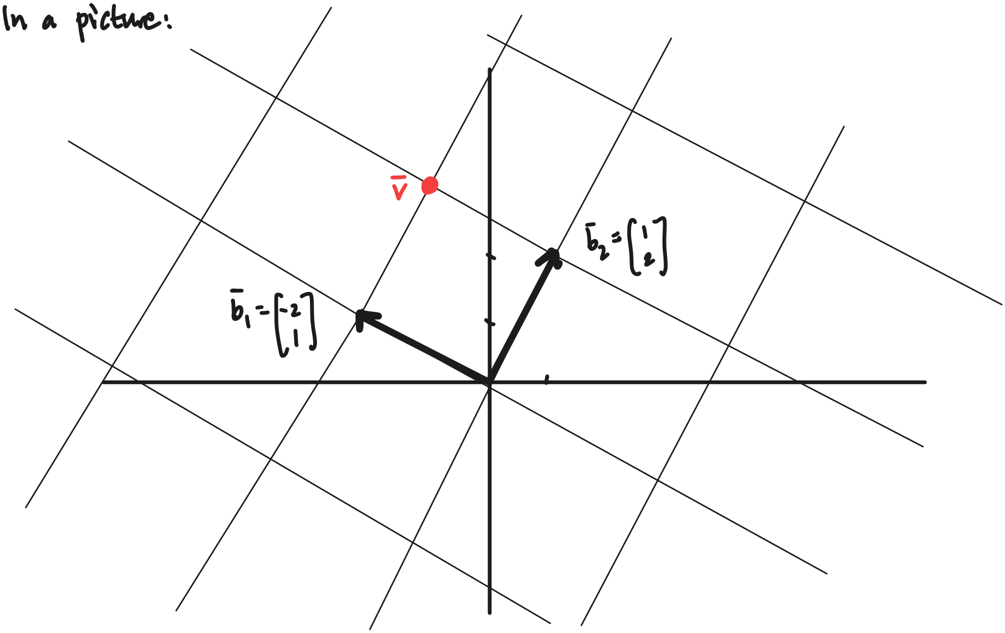
ie. the diagonal matrix D is $[\bar{x} \mapsto A\bar{x}]_{\mathcal{B}}$

A-eigenvector basis

$$A \quad = \quad P \quad D \quad P^{-1}$$

Ex $\begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & \\ & 8 \end{bmatrix}\begin{bmatrix} -2/5 & 1/5 \\ 1/5 & 2/5 \end{bmatrix}$

In a picture:



① $P^{-1}\bar{v}$ says: think in terms of coord system (grid) formed by $\bar{b}_1, \bar{b}_2$

$$\begin{bmatrix} -2/5 & 1/5 \\ 1/5 & 2/5 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

② $DP^{-1}\bar{v} = D[\bar{v}]_{\mathcal{B}}$ say scale according to eigenvalues. (easy to compute)

$$D[\bar{v}]_{\mathcal{B}} = \begin{bmatrix} 3 & \\ & 8 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$$

a.k.a.
$D(\bar{b}_1 + \bar{b}_2) = D\bar{b}_1 + D\bar{b}_2 = 3\bar{b}_1 + 8\bar{b}_2$

③ $PDP^{-1}\bar{v}$ says translate answer back to std coord system

$$PDP^{-1}\bar{v} = P\begin{bmatrix} 3 \\ 8 \end{bmatrix} = 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} + 8\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 19 \end{bmatrix}$$ (draw)