

Math 200 - Linear Algebra

Name:

Exam 1: Practice Exam

Please be sure to neatly **show and explain all of your work** and clearly label your answers. This exam is a closed-book, closed-notebook exam. Calculators are not allowed.

Please write and sign the Honor Pledge here when you are done:

Signed:

Problem	Points
1	/12
2	/12
3	/10
4	/12
5	/12
6	/12
7	/10
Total	/80

1. (a) Suppose that a vector \bar{b} can be written as a linear combination of the vectors

$$\begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 2 \\ 9 \end{bmatrix}.$$

Is the linear combination unique? In other words, is there only one way that the \bar{b} can be written as a linear combination of these vectors?

- (b) Can every vector in \mathbb{R}^4 be written as a linear combination of the vectors above? Justify your response.

2. Suppose that

$$A = \begin{bmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ 0 & -3 & -6 \end{bmatrix}$$

Express the solution set of $A\bar{x} = \bar{0}$ in parametric vector form.

(Note: A closely related question would be to find a collection of vectors that span the solution set of $A\bar{x} = \bar{0}$.)

3. Please mark each statement as True or False. (Recall that a statement is true if it holds in *all* cases. If the statement fails to hold even once, it is false.) Justify your answer: if the statement is true, give a brief explanation. If the statement is false, give an example that shows that it is false.
- (a) For an $m \times n$ matrix A , the columns $\bar{a}_1, \dots, \bar{a}_n$ of A are linearly dependent if and only if the matrix equation $A\bar{x} = \bar{0}$ has a non-trivial solution.

- (b) If $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ is a linearly dependent set, then $\bar{v}_2 = c\bar{v}_1 + d\bar{v}_3$ for some scalars c and d .

4. Suppose that A is an 5×5 matrix and that the linear transformation $\bar{x} \mapsto A\bar{x}$ is one-to-one. Please give a direct argument, based on attributes of A , that explains why the map $\bar{x} \mapsto A\bar{x}$ is onto. As part of your explanation, make sure to say what it means for a linear transformation to be one-to-one and what it means for a linear transformation to be onto.

(Note: please give a full explanation. A simple reference to the Invertible Matrix Theorem will not suffice here.)

5. Suppose that A is a 4×3 matrix. Suppose that three vectors \bar{v}_1 , \bar{v}_2 , and \bar{v}_3 in the solution set of the equation $A\bar{x} = \bar{0}$ span \mathbb{R}^3 . Write a short paragraph that justifies why $A\bar{y} = \bar{0}$ for all $\bar{y} \in \mathbb{R}^3$.

(Hint: To begin, what does it mean for a vector \bar{v} to be in the solution set of the equation $A\bar{x} = \bar{0}$? Now, suppose that $\bar{y} \in \mathbb{R}^3$. How can you relate \bar{y} to \bar{v}_1 , \bar{v}_2 , and \bar{v}_3 ?)

6. Consider the map $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1^2 \\ x_1 + x_2 \end{bmatrix}$$

Is T a linear transformation? Either prove that it is or give an example that shows that it is not, using the **definition of a linear transformation**.

7. Compute the inverse of the matrix

$$\begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & 0 \end{bmatrix}.$$

(As extra practice, you could multiply your answer by the matrix above to check your work.)