Math 200 - Linear Algebra Name:

Exam 2: Practice Exam

Please be sure to neatly **show and explain all of your work** and clearly label your answers. This exam is a closed-book, closed-notebook exam. Calculators are not allowed.

Please write and sign the Honor Pledge here when you are done:

Signed:

Problem	Points .
1	/10
2	/12
3	/12
4	/12
5	/12
6	/12
7	/10
Total	/80

1. Suppose that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is the linear transformation given by $\bar{x} \mapsto A\bar{x}$ where

$$A = \begin{bmatrix} 5 & 1 & 3 \\ 0 & 0 & 2 \\ -4 & -2 & 7 \end{bmatrix}.$$

Let S be a subset of $\mathbb{R}^3$ of volume 6. What is the volume of $T(S)$?

2. Consider the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$$

for $\mathbb{R}^2$.

(a) Compute the change of basis matrix that transforms vectors written in the standard basis of $\mathbb{R}^2$ to $\mathcal{B}$-coordinate vectors.

(b) Suppose that $\bar{v} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$. What is $[\bar{v}]_{\mathcal{B}}$.

3. Suppose that D is a fixed 4×4 matrix. Let H be the set of all 4×4 matrices that commute with D. In other words,

$$H = \{A \in M_{4\times 4} \,|\, AD = DA\}.$$

Prove that H is a subspace of $M_{4\times 4}$.

(Note: the matrix D here is unchanging. We use it to define the set H, but the elements you want to consider for this problem are the matrices A such that $AD = DA$.)

4. Are the following elements of $\mathbb{P}_2$ linearly independent?

$$\{1 + t, 3 + 2t + 3t^2, 1 + 3t^2\}$$

Please show all work and fully explain your reasoning.

5. Please mark the following statements true or false. If a statement is always true, give a brief justification why it is true. If it is false, give an example that shows that it is false.

(a) If V contains a collection $\{\bar{v}_1, \ldots, \bar{v}_k\}$ that are linearly independent, then $\dim V = k$.

(b) The map $T : M_{2\times 2} \to \mathbb{R}$ given by

$$T(A) = \det A$$

is a linear transformation.

(c) For the linear transformation $T : \mathcal{F}(\mathbb{R}; \mathbb{R}) \to \mathbb{R}^3$ given by

$$T(f) = \begin{bmatrix} f(1) \\ f(0) \\ f(1) \end{bmatrix},$$

the kernel of T is the set of all functions f such that $f(0) = f(1) = 0$.

6. Consider the map $T : \mathbb{P}_3 \to \mathbb{P}_3$ given by

$$T(\bar{p}) = \bar{p}'$$

where $\bar{p}'$ is the first derivative of the polynomial $\bar{p}$.

(For example, if $\bar{p}(t) = 1 + 3t + 5t^2 + t^3$, then $\bar{p}'(t) = 3 + 10t + 3t^2$.)

Note that T is a linear transformation (you do not need to prove this). Please find a basis for the kernel of T.

Hint: begin by finding a description of the elements of the kernel of T.

7. Suppose A is a 5×3 maxtrix. Suppose that for each $\bar{b}$ for which $A\bar{x} = \bar{b}$ has a solution, the solution set to $A\bar{x} = \bar{b}$ is infinite.

   (a) What are the possible values for the rank of A? Please justify your response.

   (b) What are the possible dimensions for $\operatorname{Nul} A$. Please justify your response.