Math 200 - Linear Algebra Name:

Final: Practice Exam

Please be sure to neatly **show and explain all of your work** and clearly label your answers. This exam is a closed-book, closed-notebook exam. Calculators are not allowed.

Please write and sign the Honor Pledge here when you are done:

Signed:

Problem	Points .
1	/12
2	/12
3	/10
4	/12
5	/12
6	/10
7	/12
Total	/80

Note: This practice test covers only material that we have covered since the second midterm. To review material from earlier in the semester, please use your midterms and practice midterms.

1. Is the following matrix A diagonalizable? If so, give P and D such that $A = PDP^{-1}$. If not, explain why not.

$$A = \begin{bmatrix} 1 & 1 \\ -4 & 5 \end{bmatrix}$$

2. For each of the following, find the matrix of the linear transformation T relative to the basis $\mathcal{B}$ for the domain and $\mathcal{F}$ for the codomain.

   (a) $T : \mathbb{R}^2 \to M_{2\times 2}$ given by

$$T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} 3a+b & -2a+5b \\ -a+2b & a-8b \end{bmatrix}.$$

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}, \quad \mathcal{F} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(\bar{x}) = \bar{x}$.

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}, \quad \mathcal{F} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}.$$

3. Please mark the following true or false. If the statement is true, give a short explanation why it is true. If it is false, give a counterexample that shows that it is false.

   (a) For a noninvertible $n \times n$ matrix A, the 0-eigenspace of A is the same as $\operatorname{Nul} A$.

   (b) Suppose that $\{\bar{v}_1, \dots, \bar{v}_p\}$ is a basis for a subspace W of $\mathbb{R}^n$. If $\bar{y} \in \mathbb{R}^n$, then

$$\operatorname{proj}_W(\bar{y}) = \left(\frac{\bar{y} \cdot v_1}{v_1 \cdot v_1}\right)\bar{v}_1 + \cdots + \left(\frac{\bar{y} \cdot v_p}{v_p \cdot v_p}\right)\bar{v}_p.$$

4. Suppose a basis for a subspace W of $\mathbb{R}^4$ is given by

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \end{bmatrix} \right\}.$$

Find an orthogonal basis for W.

5. Consider the system $A\bar{x} = \bar{b}$ where

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 5 \\ 2 & 0 \end{bmatrix}, \quad \bar{b} = \begin{bmatrix} 21 \\ 1 \\ 2 \end{bmatrix}.$$

Notice that this system is inconsistent. Find all least-squares solutions of the system.

6. Use dot products to prove the Pythagorean Theorem, which says that for $\bar{u}, \bar{v} \in \mathbb{R}^n$,

$$\|\bar{u} + \bar{v}\|^2 = \|\bar{u}\|^2 + \|\bar{v}\|^2$$

if and only if $\bar{u} \perp \bar{v}$.

7. The matrix

$$A = \begin{bmatrix} 4 & 0 & 2 \\ 2 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix}.$$

has eigenvalues 3 and 4. Find a basis for the 3-eigenspace of A. Is A diagonalizable?