Math 200: Linear Algebra
Final Exam Review

This is a list of topics for our final exam. It is not necessarily exhaustive, but it covers all of the main ideas that we have seen since this semester. I hope this list will help you develop a broad overview of our course so that you can consolidate and integrate what you've learned these last few months.

When studying, you can use homework problems and/or other problems from the textbook as practice to help solidify your understanding. Try for a blend of computational and theoretical practice. To practice the theory, in addition to proof-type problems from the book it's a great idea to review the proofs that we have done in class. In general, if there is a particular concept or area that you find challenging, you might consider focusing your attention on problems related to that topic. I also highly recommend reviewing both midterms.

- The Vector Space $\mathbb{R}^n$
  - Systems of equations, vector equations, and matrix equations: how is everything related?
  - Linear independence and span: definitions and interpretations.
  - Solutions sets of linear equations.
    - Row reduction.
    - When (and why) does $A\bar{x} = \bar{0}$ have a nontrivial solution?
    - When (and why) does $A\bar{x} = \bar{b}$ have a solution for all $\bar{b}$?
  - Linear transformations $\mathbb{R}^n \to \mathbb{R}^m$.
    - Definition of a linear transformation.
    - Standard matrix of a linear transformation.
    - What is Nul$A$? How do you compute it?
    - What is Col$A$? How do you compute it?
  - Matrix inverses: How to compute and the Invertible Matrix Theorem.

- Determinants
  - How to compute, row/column expansion.
  - Algebraic properties of determinants.
  - $\det A \neq 0$ if and only if A is invertible (why is this true?).

- Vector Spaces in General
  - Definition of vector space.
  - Subspaces: definition and how to prove that a subset $H \subset V$ is a subspace.
  - Special subspaces related to a matrix A: $\text{Nul}A, \text{Col}A$....definitions, computation, and bases for these spaces.
  - Linear transformations: definitions, proving a function is a linear transformation. (How is this related to the definition of a vector space?).
  - Definition of a function. Definitions of one-to-one and onto, and how one would show that a function is one-to-one or onto.
  - Definitions of linear independence and dependence.
  - Bases: definition. What are the implications of the two components of the definition and how do they balance each other?
  - Coordinate mapping/systems: definition and importance/significance.
  - Dimension. Spanning set theorem and basis theorem.
  - Rank: definition, Rank-Nullity theorem and a geometric interpretation.
  - Change of basis: how to compute and what it means.
  - The matrix of a linear transformation $T : V \to W$ relative to bases $\mathcal{B}$ and $\mathcal{F}$.

- Eigenvalues and Eigenvectors of a Matrix A
  - How to compute and interpret eigenvalues and eigenvectors.
  - Under what conditions is a matrix A diagonalizable?
  - How to diagonalize a diagonalizable matrix.
  - Interpretations of the equation $A = PDP^{-1}$.
  - How does diagonalization relate to the matrix of a linear transformation?

- Orthogonality and Least Squares
  - Dot product in $\mathbb{R}^n$.
  - How to compute length, distance, angles, orthogonality using a dot product.
  - Orthogonal/orthonormal sets and bases.
  - Orthogonal Decomposition Theorem.
  - Gram-Schmidt process.
  - Least squares method and applications to linear models.