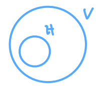
Summary of Vector Spaces

- $\mathbb{R}^n$ is an example of a vector space.

  ↳ other examples include $\mathbb{P}_n$, $M_{m \times n}$, and $\mathcal{F}(\mathbb{R};\mathbb{R})$

- Vector spaces contain subspaces.



  ↳ examples include $\{\bar{0}_V\}$ and, for a given matrix A, Nul A and Col A.

- A linear transformation is a map $T: V \to W$ b/w vector spaces that respects vector space operations.

- The definitions of linear independence and span carry to general vector spaces. Thus we can define a basis for a vector space.

- The dimension of a vector space is given by the number of elements in a basis for the vector space.

- Given an n-dim'l vector space V with basis $\mathcal{B}$, we can represent any vector $v \in V$ by a $\mathcal{B}$-coordinate vector $[\bar{v}]_{\mathcal{B}} \in \mathbb{R}^n$.

- Given an n-dim'l vector space V with basis $\mathcal{B}$ and a p-dim'l vector space W with basis $\mathcal{F}$, we can represent any LT $T: V \to W$ by a matrix ${}_{\mathcal{F}}[T]_{\mathcal{B}}$.

  the std matrix of a LT (section 1.7)

  change of coords matrix in $\mathbb{R}^n$ $\mathcal{B} \to \mathcal{E}$ (section 4.4)

  ↳ examples include $A = {}_{\mathcal{E}}[T]_{\mathcal{E}}$, $P_{\mathcal{B}} = {}_{\mathcal{E}}[Id]_{\mathcal{B}}$,

  change of basis matrix in V (section 4.6)

  and ${}_{\mathcal{C} \leftarrow}P_{\mathcal{B}} = {}_{\mathcal{C}}[Id]_{\mathcal{B}}$.

- Given an $n \times n$ matrix A, if $T: \mathbb{R}^n \to \mathbb{R}^n$ is given by $\bar{x} \mapsto A\bar{x}$, when there is a basis $\mathcal{B}$ for $\mathbb{R}^n$ of eigenvectors of A, then ${}_{\mathcal{B}}[T]_{\mathcal{B}} = D$, a diagonal matrix.