MATH 200C: Linear Algebra

Class 19: Monday, March 30, 2026


Welcome Back
from Spring
Break!

Notes on Assignment 17

Notes on Project 1 Proofs

Vector Spaces

Exam 2: Wednesday, April 8

Vector Spaces

Vector Spaces and Subspaces

Two Examples: $\mathbb{R}^4$ and S, the set of all 2×3 matrices.
Arithmetic Operations: Addition and Scalar Multiplication.
All Simple Rules of Arithmetic Hold.

Definition: A **vector space** is a nonempty set V of objects, called vectors, on which are defined two operations, called **addition** and **multiplication by scalars** (real numbers), subject to the ten axioms (or rules) listed below.
The axioms must hold for all vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in V and for all scalars c and d.

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The Axioms

1. The sum of **u** and **v**, denoted by **u** + **v**, is in *V*.	6. The scalar multiple of **u** by *c*, denoted by *c***u**, is in *V*
2. **u** + **v** = **v** + **u**.	7. c(**u** + **v**) = *c***u** + *c***v**
3. (**u** + **v**) + **w** = **u** + (**u** + **w**)	8. (*c* + *d*) /**u** = *c***u** + *d***u**
4. There is a zero vector **0** in *V* such that **u** + **0** = **u**	9. *c*(*d***u**) = (*cd*)**u**.
5. For each **u** in *V*, there is a vector -**u** in *V* such that **u** + -**u** = **0**	10. 1**u** = **u**

Some Examples of Vector Spaces

- $\mathbb{R}^n$, for $n \geq 1$
- All 3 $\times$ 5 matrices
- All $m \times n$ matrices for a fixed m and n.
- All arrows in 3 dimensional space
- All infinite sequences of real numbers $a_1, a_2, a_3, \ldots$
- $\mathbb{P}_n$, polynomials of degree $\leq n$
- $\mathbb{P}$, polynomials
- Real-Values functions on an interval I.

Some Simple Consequences of Axioms

For each $\mathbf{u}$ in V and scalar c,

$$0\mathbf{u} = \mathbf{0}$$

$$c\mathbf{0} = \mathbf{0}$$

$$-\mathbf{u} = (-1)\mathbf{u}$$

Subspaces

In many problems, a vector space consists of an appropriate subset of vectors from some larger vector space.
In this case, only three of the ten vector space axioms need to be checked; the rest are automatically satisfied.
Definition: A **subspace** of a vector space V is a subset H of V that has three properties:

(a) The zero vector of V is in H

(b) H is closed under vector addition. That is, for each $\mathbf{u}$ and $\mathbf{v}$ in H , the sum $\mathbf{u}+\mathbf{v}$ is in H

(c) H is closed under multiplication by scalars. That is, for each $\mathbf{u}$ in H and each scalar c, the vector $c\mathbf{u}$ is in H .

Subspaces are closed under addition and scalar multiplication.
Thus they are closed under **Linear Combinations**
We can talk about **Linear Indepedence** and **Span**

Theorem 1:

If $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p}$ are in a vector space V,
then span $(\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_p})$ is a subspace of V.