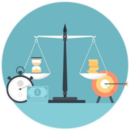
MATH 200C: Linear Algebra

Balancing Chemical Equations



Class 8: February 25, 2026

- Notes on Assignment 6
- Linear Independence

Announcements

Exam 1: Next Wednesday, 7 PM - No Time Limit

No Books,Computers, Smart Phones, etc.

One Page of Your Own Notes OK

Section 1.6: Applications of Linear Equations

- **Leontief Input Output Models**
  - Price Equilibrium (Text Example)
  - Meeting External Demand (Class Example)
- **Balancing Chemical Reaction Equations**
- **Network Flow Problems**

Exercise 3 from Section 1.6

Exchange Matrix E

Chemicals	Fuels	Machinery	Purchased From	Price
.10	.80	.40	Chemicals	p_C
.30	.10	.40	Fuels	p_F
.50	.10	.20	Machinery	p_M

Machinery sells 40% of its output to Chemistry, another 40% to Chemicals and retains the rest (10%)

Entry in row i and column j is percentage of output of sector j sold to sector i

Sector i buys that percentage of sector j's output.

Chemistry	Receives p_C	Pays $.10\ p_C + .80\ p_F + .40\ p_M$
Fuels	Receives p_F	Pays $.30\ p_C + .10\ p_F + .40\ p_M$
Machinery	Receives p_M	Pays $.50\ p_C + .\ 10\ p_F + .20\ p_M$

Price Equilibrium

Chemistry	Receives p_C	Pays $.10\ p_C + .80\ p_F + .40\ p_M$
Fuels	Receives p_F	Pays $.30\ p_C + .10\ p_F + .40\ p_M$
Machinery	Receives p_M	Pays $.50\ p_C + .\ 10\ p_F + .20\ p_M$

$$\begin{aligned} .10\ p_C + .80\ p_F + .40\ p_M &= p_C \\ .30\ p_C + .10\ p_F + .40\ p_M &= p_F \\ .50\ p_C + .\ 10\ p_F + .20\ p_M &= p_M \end{aligned}$$

$$E \begin{bmatrix} p_C \\ p_F \\ p_M \end{bmatrix} = \begin{bmatrix} p_C \\ p_F \\ p_M \end{bmatrix}, E\mathbf{p} = \mathbf{p} \text{ where } \mathbf{p} = \begin{bmatrix} p_C \\ p_F \\ p_M \end{bmatrix}$$

Input –– Output Economics: Meeting External Demand

Input Required Per Dollar Output

consumption matrix C for the economy, whose columns are the imports required for each unit of output, we obtain $C\mathbf{x}$, the portion of the production vector that will be consumed by the productive sectors. We call the vector $C\mathbf{x}$ is called the **intermediate demand vector** for the economy.

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Input Required Per Dollar Output

$$C = \begin{bmatrix} .5 & .1 & .1 \\ .2 & .5 & .3 \\ .1 & .3 & .4 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/10 & 1/10 \\ 1/5 & 1/2 & 3/10 \\ 1/10 & 3/10 & 2/5 \end{bmatrix}, \mathbf{d} = \begin{bmatrix} 7900 \\ 3950 \\ 1975 \end{bmatrix}$$

Find **x** so that **x** - C **x** = **d**

Price Equilibrium	Meet External Demand
Find $\mathbf{p}$ so that	Find $\mathbf{x}$ so that
$E\mathbf{p} = \mathbf{p}$	$\mathbf{x} - C\mathbf{x} = \mathbf{d}$

Balancing Chemical Reactions

When solutions of Sodium Phosphate and Barium Nitrate are mixed, the result is
Barium Phosphate (as a precipitate) and Sodium Nitrate

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Unbalanced Equation is

$$Na_3PO_4 + Ba(NO_3)_2 \rightarrow Ba_3(PO_4)_2 + NaNO_3$$

where

Na = Sodium

P = Phosphorous

O = Oxygen

Ba = Barium

N = Nitrogen

$$Na_3PO_4 + Ba(NO_3)_2 \rightarrow Ba_3(PO_4)_2 + NaNO_3$$

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$$\begin{pmatrix} Na \\ P \\ O \\ Ba \\ N \end{pmatrix} : \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix} x_2 = \begin{bmatrix} 0 \\ 2 \\ 8 \\ 3 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix} x_4$$

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Row Reduced Echelon Form is

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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General Solution is

$$\begin{aligned} x_1 &= (1/3)x_4 \\ x_2 &= (1/2)x_4 \\ x_3 &= (1/6)x_4 \\ x_4 &\ \text{free choice} \end{aligned}$$

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Simplest Solution is:
$$\begin{aligned} x_1 &= 2 \\ x_2 &= 3 \\ x_3 &= 1 \\ x_4 &= 6 \end{aligned}$$

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Balanced Equation is

$$2Na_3PO_4 + 3Ba(NO_3)_2 \rightarrow Ba_3(PO_4)_2 + 6NaNO_3$$