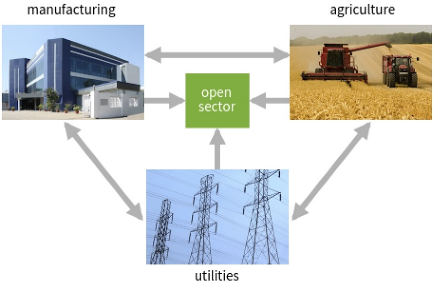
MATH 200C: Linear Algebra



Class 7: February 23, 2026

Handouts

- Notes on Assignment 5
- Leontief Input – Output Example

Announcements

Exam 1: Next Wednesday, 7 PM - No Time Limit

No Books,Computers, Smart Phones, etc.

One Page of Your Own Notes OK

Section 1.6: Applications of Linear Equations

- Leontief Input Output Models
- Balancing Chemical Reaction Equations
- Network Flow Problems

$$2x + 3y + 5z = 203$$
$$x + y + z = 51$$
$$2x + 9z = 26$$
$$5x + 4y + 15z = 492$$

$$\begin{bmatrix} 2 & 3 & 5 & | & 203 \\ 1 & 1 & 1 & | & 51 \\ 2 & 0 & 9 & | & 26 \\ 5 & 4 & 15 & | & 492 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 2 \\ 0 & 1 & 0 & | & 23 \\ 0 & 0 & 1 & | & 26 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \quad \text{so} \quad \begin{aligned} x &= 2 \\ y &= 23 \\ z &= 26 \end{aligned}$$

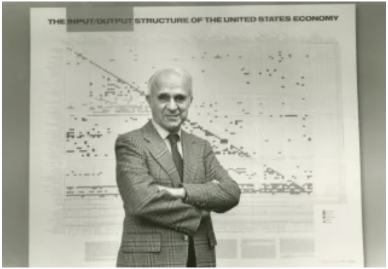
$$\text{3 by 3 Identity Matrix } \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$3 \text{ by } 3 \text{ Identity Matrix } \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **MATLAB: I = eye(3)**

$$4 \text{ by } 4 \text{ identity Matrix } \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ **MATLAB: I = eye(4)**

$$2 \text{ by } 2 \text{ Identity Matrix } \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ **MATLAB: I = eye(2)**

$$1 \text{ by } 1 \text{ Identity Matrix } \mathbf{I} = \begin{bmatrix} 1 \end{bmatrix}$$ **MATLAB: I = eye(1)**



Wassily Leontief
Born Vasily Vasilyevich Leontiev
August 5, 1905 – February 5, 1999
Nobel Memorial Prize in Economic Sciences (1973)

The **Open Sector** of an economy (the sector that does not produce outputs) wants the economy to supply it with goods, products, and utilities with monetary values.
We call the column vector $\mathbf{d}$ that has these numbers as components the **outside demand vector**.
Since the product -producing sectors consume some of their own output, the monetary value of their output must cover their own needs plus the outside demand.
The column vector $\mathbf{x}$ that has these monetary value numbers as components is called the **production vector** for the economy.
By multiplying $\mathbf{x}$ by the **consumption matrix** C for the economy, whose columns are the imports required for each output, we obtain $C\mathbf{x}$, the portion of the production vector that will be consumed by the productive sectors. We call the vector $C\mathbf{x}$ is called the **intermediate demand vector** for the economy.
Thus $\mathbf{x}$ must satisfy the equation $\mathbf{x} - C\,\mathbf{x} = \mathbf{d}$

Input –– Output Economics

Input Required Per Dollar Output

	Manufacturing	*Agriculture*	*Utilities*
Manufacturing	.5	.1	.1
Agriculture	.2	.5	.3
Utilities	.1	.3	.4

Input — Output Economics

Input Required Per Dollar Output

$$C = \begin{bmatrix} .5 & .1 & .1 \\ .2 & .5 & .3 \\ .1 & .3 & .4 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/10 & 1/10 \\ 1/5 & 1/2 & 3/10 \\ 1/10 & 3/10 & 2/5 \end{bmatrix}, \mathbf{d} = \begin{bmatrix} 7900 \\ 3950 \\ 1975 \end{bmatrix}$$

Find **x** so that **x** - C **x** = **d**

Write as **d** =I **x** - C **x** = (I - C)**x**

Solve system of linear equations: A **x** = **d** where A = I - C

I - C =

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1/2 & 1/10 & 1/10 \\ 1/5 & 1/2 & 3/10 \\ 1/10 & 3/10 & 2/5 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/10 & -1/10 \\ -1/5 & 1/2 & -3/10 \\ -1/10 & -3/10 & 3/5 \end{bmatrix}$$

Solve A $\mathbf{x} = \mathbf{d}$ where

$$A = \begin{bmatrix} 1/2 & -1/10 & -1/10 \\ -1/5 & 1/2 & -3/10 \\ -1/10 & -3/10 & 3/5 \end{bmatrix} \text{ and } \mathbf{d} = \begin{bmatrix} 7900 \\ 3950 \\ 1975 \end{bmatrix}$$

Augmented Matrix is

$$\begin{bmatrix} 1/2 & -1/10 & -1/10 & | & 7900 \\ -1/5 & 1/2 & -3/10 & | & 3950 \\ -1/10 & -3/10 & 3/5 & | & 1975 \end{bmatrix}$$

which row reduces to

$$\begin{bmatrix} 1 & 0 & 0 & | & 27500 \\ 0 & 1 & 0 & | & 33750 \\ 0 & 0 & 1 & | & 24750 \end{bmatrix}$$

Balancing Chemical Reactions

When solutions of Sodium Phosphate and Barium Nitrate are mixed, the result is
Barium Phosphate (as a precipitate) and Sodium Nitrate

Unbalanced Equation is

$$Na_3PO_4 + Ba(NO_3)_2 \rightarrow Ba_3(PO_4)_2 + NaNO_3$$

where

$$\begin{aligned} Na &= \text{Sodium} \\ P &= \text{Phosphorous} \\ O &= \text{Oxygen} \\ Ba &= \text{Barium} \\ N &= \text{Nitrogen} \end{aligned}$$

$$Na_3PO_4 + Ba(NO_3)_2 \to Ba_3(PO_4)_2 + NaNO_3$$

$$\begin{pmatrix} Na \\ P \\ O \\ Ba \\ N \end{pmatrix} : \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix} x_2 = \begin{bmatrix} 0 \\ 2 \\ 8 \\ 3 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix} x_4$$

$$\begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix} x_2 - \begin{bmatrix} 0 \\ 2 \\ 8 \\ 3 \\ 0 \end{bmatrix} x_3 - \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix} x_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & -1 \\ 1 & 0 & -2 & 0 \\ 4 & 6 & -8 & -3 \\ 0 & 1 & -3 & 0 \\ 0 & 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & -1 \\ 1 & 0 & -2 & 0 \\ 4 & 6 & -8 & -3 \\ 0 & 1 & -3 & 0 \\ 0 & 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 0 & 0 & -1 \\ 1 & 0 & -2 & 0 \\ 4 & 6 & -8 & -3 \\ 0 & 1 & -3 & 0 \\ 0 & 2 & 0 & -1 \end{bmatrix}$$

Row Reduced Echelon Form is

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row Reduced Echelon Form is

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -1/6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

General Solution is
$$\begin{aligned} x_1 &= (1/3)x_4 \\ x_2 &= (1/2)x_4 \\ x_3 &= (1/6)x_4 \\ x_4 &\ \text{free choice} \end{aligned}$$

Simplest Solution is:
$$\begin{aligned} x_1 &= 2 \\ x_2 &= 3 \\ x_3 &= 1 \\ x_4 &= 6 \end{aligned}$$

Balanced Equation is

$$2Na_3PO_4 + 3Ba(NO_3)_2 \rightarrow Ba_3(PO_4)_2 + 6NaNO_3$$