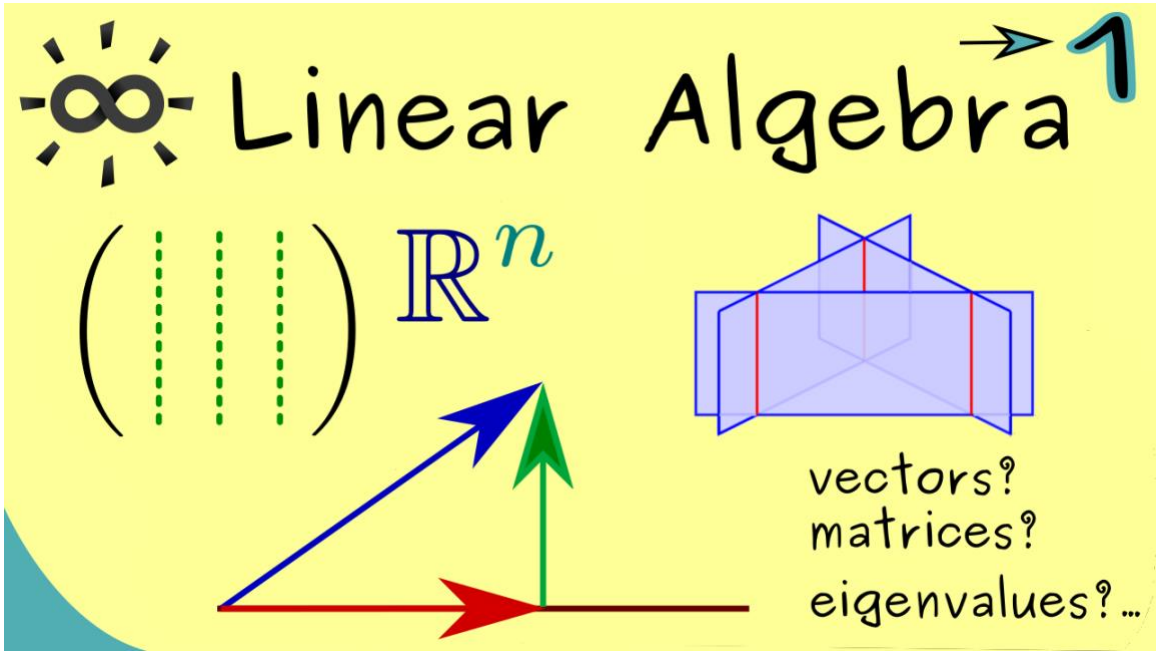
MATH 200C
LINEAR ALGEBRA

Michael Olinick



Spring 2026

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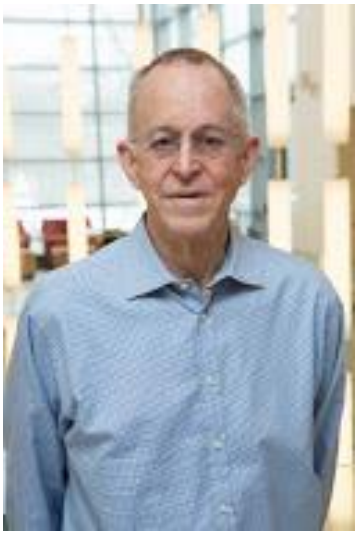
Excerpts from "The Growing Importance of Linear Algebra in Undergraduate Mathematics" by Alan Tucker

Linear algebra stands today as the epitome of accessible, yet powerful mathematical theory. Linear algebra has many appealing facets which radiate in different directions… A mastery of finite vector spaces, linear transformations, and their extensions to function spaces is essential for a practitioner or researcher in most areas of pure and applied mathematics. Linear algebra is the mathematics of our modern technological world of complex multivariable systems and computers.

… While mathematical methods – principally calculus-based analysis - were once largely restricted to the physical sciences, tools of linear algebra find use in almost all academic fields and throughout modern society. The interaction with modern computation is especially appealing: previously, theory was needed to give analytic answers since explicit computation was hopelessly tedious; nowadays, theory is used to guide increasingly complex computations…

There is an even more pervasive practical side of linear algebra. Stated in starkest terms, linear problems are solvable while nonlinear problems are not. Of course, some nonlinear problems with a small number of variables can be solved, but 99.99% of multivariable nonlinear problems can be solved only by recasting them as linear systems.

The theoretical status of linear algebra is as important as its applicability and its role in computation. Vector spaces and linear transformations are central themes of much of mathematics. The fact that differentiation is a linear operator lies at the heart of the power of calculus and differential equations.



***Alan Tucker is Distinguished Teaching Professor at SUNY Stony Brook*.**

He is a Fellow of the American Mathematical Society and of the American Association for the Advancement of Science. He is editor-in-chief of *Applied Mathematical Letters* and author of the textbook *Applied Combinatorics*, now in its 6th edition.

The Growing Importance of Linear Algebra in Undergraduate Mathematics
The College Mathematics Journal, Vol. 24, No. 1 (January 1993), pp. 3-9

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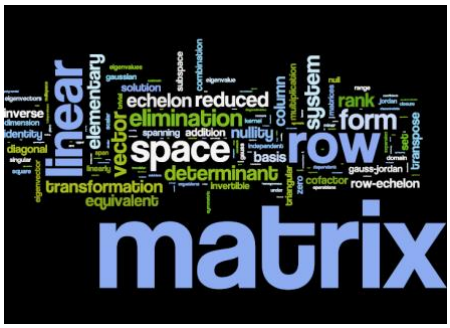


The Growing Importance of Linear Algebra in Undergraduate Mathematics (Excerpt)

MATH 200C : Linear Algebra

Course Description/Syllabus
Spring Term 2026

***Course Title*:** **Linear Algebra**

***Catalog Description*:** Matrices and systems of linear equations, the Euclidean space of three dimensions and other real vector spaces, independence and dimensions, scalar products and orthogonality, linear transformations and matrix representations, eigenvalues and similarity, determinants, the inverse of a matrix and Cramer's rule..

Additional Description: Linear Algebra is the first mathematics course that goes beyond the basic tools of mathematics: arithmetic, algebra, geometry, trigonometry, and calculus. In addition to studying many important mathematical results and techniques, you will give much greater attention to learning definitions carefully, stating theorems precisely, and understanding the proofs of many theorems. The material covered in linear algebra plays an essential role in all of the rest of higher mathematics and related areas of computer science, statistics, physics, and other natural and social sciences.

Linear algebra is the mathematical backbone for many Nobel Prize-winning discoveries in Physics and Economic Sciences. These prizes often recognize the application of matrices, vector spaces, and linear algorithms to analyze complex systems, predict economic behaviors, and model physical realities. Google became a multi-billion dollar enterprise because of a linear algebra algorithm

We can often model real world situations at least approximately by linear relations. Whether they take place in the physical, biological, chemical, economic, political or social sphere, studying these phenomena frequently leads to examining collections of linear equations.

We start with systems of linear equations. There are a number of natural questions: When does a solution exist? Can there be multiple solutions? How do you find them? What conditions guarantee that there is a unique solution? Is there a geometric interpretation?

We will develop the theoretical structures that lead to answering such questions. Our approach will balance theory, computation and applications while helping you improve your intellectual sophistication, understanding proofs, and writing mathematical arguments.

I hope you find the course to be an intellectually stimulating and enjoyable experience!

Course Website: ***s26.middlebury.edu/MATH0200C***

Prerequisites: A good background is a year's study of calculus in secondary school or completion of MATH 121:Calculus I at Middlebury. Very little in the way of technical knowledge of differentiation and integration techniques will be required. In fact, students with no calculus background but four years of secondary school mathematics can excel in Linear Algebra. What is needed is curiosity, a thirst for understanding, good organizational skills, and an appreciation for the importance of clarity and precision in language.

Learning Objectives:

- Understand basic linear algebra techniques
- Gain the skills to perform computations involving vectors, matrices, and systems of linear equations
- Appreciate applications of linear algebra to economics, biology, computer sciences, physics, engineering, the social sciences, etc
- Strengthen your desire for further study of mathematics
- Improve your ability to write a logical and coherent mathematical proof

***Instructor*:** Michael Olinick, Office: 202 Warner, Phone: 443-5559. Home telephone: 388-4290; email: *molinick@middlebury.edu*. My usual office hours will be Monday, Wednesday and Friday from 8:30 to 9:30 and 10:40 to 11 AM, and 12:05 to 1 PM on Monday and Wednesday. I would be happy to make an appointment to see you at other mutually convenient times.

***Meeting Times*:** MATH 200C: MWF 11:15AM – 12:05 PM (Warner 100)

***Prerequisites*:** Calculus I (MATH 121) or permission.

***Textbook*:** David Lay, Steven Lay and Judi McDonald, *Linear Algebra And Its Applications,* Sixth Edition: Pearson, 2020 update; ISBN-13: 9780135851258. Your daily assignments will include a few pages of reading in the text. Be certain to read the book carefully (with pencil and paper close by!) and to complete the relevant reading **before** coming to class and before embarking on the homework problems.

Computer Algebra Systems: Mathematically oriented software such as *MATLAB*, *Maple*, and *Mathematica* give you an opportunity to investigate the ideas of linear algebra in ways not available to previous generations of students. Relatively simple commands can direct a computer to carry out complex calculations rapidly and without error. More importantly, you can create and carry out experiments to develop and test your own conjectures. The very powerful graphics capabilities of these applications provide you with strong tools to deepen your understanding of Linear Algebra . *MATLAB* is an especially powerful computational and visualization tool which is used extensively in scientific research and engineering applications. Intermediate and advanced required and elective courses in Middlebury's Applied Mathematics Track adopt *MATLAB* as the de facto programming tool. There will be a required introductory *MATLAB* workshop late in the first week of the term; you will be able to choose either a Thursday evening or Friday afternoon session.

***Requirements*:** There will be three midterm examinations and a final examination in addition to required daily homework assignments and an extended independent team project. The midterm examinations will be given in the evening to eliminate time pressure. Tentative dates for these tests are:

Wednesday, March 4
Wednesday, April 8
Wednesday, May 6

Final Exam: The registrar's office has **tentatively** set the date and time of our final exam:
Thursday, May 14, 9 AM – Noon

Course Grades: Each of the midterm exams will be worth approximately 20%, the final about 30%, projects roughly 10%. I will make adjustments with later work counting more heavily if students show improvement over earlier results.

The mathematics department regards a C grade as an indication of satisfactory *understanding* of the course material, a B as good/very good understanding and an A as an excellent/superior grasp of the material. Typically, but not always, these tend to correlate with averages in the 70's, 80's and 90's, respectively. I do strive to issue course grades keeping in mind the hundreds of students I have had in calculus classes over the years.

Accommodations Students who have Letters of Accommodation in this class are encouraged to contact me as early in the semester as possible to ensure that such accommodations are implemented in a timely fashion. For those without Letters of Accommodation, assistance is available to eligible students through the Disability Resource Center (DRC). Please contact ADA Coordinators Jodi Litchfield and Peter Ploegman of the DRC at ada@middlebury.edu for more information. All discussions will remain confidential.

Homework: Mathematics is not a spectator sport! You must be a participant. The only effective way to learn mathematics is to do mathematics. In your case, this includes working out many multivariable calculus problems.

There will be daily written homework assignments which you will be expected to complete and submit. They will be corrected and assigned a numerical score, but I view these assignments primarily as **learning** rather than testing experiences. I will occasionally assign some challenging problems which everyone may not be able to solve. You should, however, make an honest attempt at every problem.

Each homework assignment will probably take you between 2 and 3 hours to complete; this includes the reading and problem solving. If you keep pace with the course by spending an hour or so each day on it, then you will be quite successful. If you wait until the end of the week and then try to spend one 6 hour block of time on the material, then experience shows you face disaster!

Honor Code: The examinations are to be taken without any information of any kind about the exam or its questions from any source but me. (I will give you a clear idea about what to expect on each exam.) Exams will be closed book, closed notes, and taken without calculators. Any

departure from these policies would be a violation of the Honor Code and thus would be subject to a judicial review.

Homework is a different story: Calculators and MATLAB are ok for checking work and for doing tedious calculations. I encourage you to discuss problems with other students. But when you do that, you must give credit for another's contributions by naming the person on your solution. Writing up each problem solution must be done by you only. Failure to follow these requirements is a violation of the Honor system. The principle here is simple: consultation and collaboration are welcome, but you must explicitly acknowledge any and all intellectual content that is not solely yours. This rule should apply in all of your college work.

Help: Please see me immediately if you have any difficulties with this course. There are ample resources on campus for assistance, including the Center for Teaching, Learning and Research in the Davis Family Library and the Q-Center ("Armstrong Quantitative Center") in BiHall. The College will be assigning tutors to work with students in our course; they will schedule drop-in help sessions several nights a week. Stay tuned for more details.

One of the essential characteristics of college life that distinguishes it from secondary school is the increased responsibility placed on *you* for your own education. **Most of what you will learn will not be told to you by a teacher inside a classroom**.

Even if our model of you were an empty vessel waiting passively to be filled with information and wisdom, there would not be time enough in our daily meetings to present and explain it all.

We see you, more appropriately, as an *active* learner ready to confront aggressively the often times subtle and difficult ideas our courses contain. You will need to listen and to read carefully, to master concepts by wrestling with numerous examples and problems, and to ask thoughtful questions.

As you progress through the undergraduate mathematics curriculum, emphasis changes from mastering techniques to solve problems to learning the theory that underlies the particular subject you are studying. *Linear Algebr* is a transitional course. You will do plenty of calculations, find many matrix products and inverses, and deal with a full quota of vector operations. You will also find more of your effort directed toward understanding definitions, statements of theorems and their proofs. You will even be expected to come up with some short proofs of your own.

One of my goals for you this term is to develop your skills in *reading* mathematical expositions. I will expect that you will have read (perhaps more than once!) in advance the sections of the text relevant to the topic we will be exploring in class that day. I will not normally present a lecture which substitutes for reading the text. I will more likely use time in class to give a broader overview or alternative proofs or interesting applications and extensions of the material or previews of the next section.

Policy on Generative AI: Please see our Policy on AI Usage Any use of generative AI tools will be treated as a violation of Middlebury's Honor Code.

MATH 200: Spring 2026
Tentative Course Outline
(Times are approximate)

Linear Equations in Linear Algebra (~ 3 weeks)

Systems of Linear Equations
Row Reduction and Echelon Forms
Vector Equations
The Matrix Equation **Ax = b**
Solution Sets of Linear Systems
Applications of Linear Systems
Linear Independence
Introduction to Linear Transformations
The Matrix of a Linear Transformation
Linear Models in Business, Science, and Engineering

Matrix Algebra (~ 3 weeks)

Matrix Operations
The Inverse of a Matrix
Characterizations of Invertible Matrices
Partitioned Matrices
Matrix Factorizations
The Leontief Input—Output Model
Applications to Computer Graphics
Subspaces of R^n
Dimension and Rank

Vector Spaces (~3 weeks)

Vector Spaces and Subspaces
Null Spaces, Column Spaces, and Linear Transformations
Linearly Independent Sets; Bases
Coordinate Systems
The Dimension of a Vector Space
Rank; Change of Basis
Applications to Markov Chains

Eigenvalues and Eigenvectors (1 week)

Eigenvectors and Eigenvalues
The Characteristic Equation
Diagonalization
Eigenvectors and Linear Transformations

Determinants

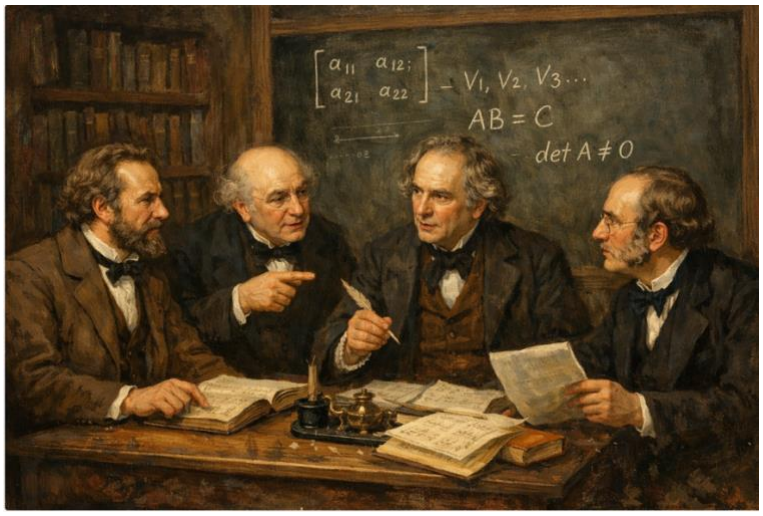
Introduction to Determinants
Properties of Determinants

Finite-State Markov Chains (1 week)

Google and Markov Chains
The Steady-State Vector and Google's PageRank

Orthogonality and Least Squares (1 week)

Inner Product, Length, and Orthogonality
Orthogonal Sets
Orthogonal Projections
The Gram—Schmidt Process
Least-Squares Problems



ChatGPT imagining Grassmann, Sylvester, Hamilton and Cayley discussing Linear Algebra

Policy on the Use of AI Tools in Linear Algebra

Purpose

Artificial Intelligence (AI) tools (such as ChatGPT, Wolfram Alpha, MATLAB's AI features, and similar systems) are increasingly common and can be valuable learning aids when used appropriately. This policy is designed to clarify **when and how AI tools may be used** in this Linear Algebra course, while maintaining academic integrity and ensuring that students develop essential mathematical reasoning skills.

Guiding Principles

1. **Learning First**: The primary goal of this course is for you to understand concepts, develop problem-solving skills, and communicate mathematical reasoning clearly. AI tools should support—not replace—your learning.
2. **Transparency**: Any permitted use of AI must be acknowledged.
3. **Responsibility**: You are responsible for understanding, verifying, and being able to explain any work you submit, regardless of whether AI was used.

Permitted Uses of AI Tools

Unless otherwise specified for a particular assignment, you may use AI tools for the following purposes: - Clarifying definitions or terminology (e.g., eigenvalues, linear independence, diagonalization). - Asking for **conceptual explanations** or alternative ways of understanding material discussed in class. - Checking computations *after* you have attempted a problem yourself. - Generating practice problems or additional examples for self-study. - Debugging code for computational assignments (e.g., MATLAB or Python), provided you understand the final code.

Prohibited Uses of AI Tools

The following uses are **not allowed** unless explicitly authorized by the instructor: - Submitting AI-generated solutions (in whole or in part) to homework, quizzes, exams, or projects as your own work. - Using AI tools during **quizzes or exams** (take-home or in-class) unless explicitly permitted. - Using AI to bypass essential reasoning steps, proofs, or explanations required in an assignment. - Copying AI-generated text, proofs, or solutions without proper acknowledgment.

Attribution and Disclosure

If you use AI tools on an assignment where their use is permitted, you must include a brief disclosure statement such as: > "I used an AI tool to help clarify concepts related to matrix diagonalization and to check intermediate computations."

Failure to disclose permitted AI use may be treated as an academic integrity violation.

Accuracy and Accountability

AI tools can make mistakes, produce incomplete reasoning, or give answers that are mathematically incorrect or inappropriate for this course. You are fully responsible for: - Verifying correctness of all results. - Ensuring that notation, methods, and explanations align with course standards. - Being able to explain your submitted work orally if asked.

Academic Integrity

Improper use of AI tools will be treated in the same manner as other forms of academic dishonesty and may result in penalties consistent with Middlebury's policies, including a zero on the assignment, failure of the course, or further disciplinary action.

Instructor Discretion

Specific assignments may have **stricter or more flexible rules** regarding AI use. Always follow the instructions given for individual assignments, even if they differ from this general policy.

Final Note

Used responsibly, AI tools can be a powerful supplement to your study of Linear Algebra. Misused, they can undermine your learning. When in doubt about whether a particular use is allowed, **ask before using the tool**

Citation for my use of AI to generate this policy

"Write a policy on the use of AI tools in a Linear Algebra Course" prompt. ChatGPT 4.0, OpenAI, January 23, 2026, chat.openai.com/chat.

More Detailed List of Topics

Linear Equations in Linear Algebra

Systems of Linear Equations
Row Reduction and Echelon Forms
Vector Equations
The Matrix Equation **Ax = b**
Solution Sets of Linear Systems
Applications of Linear Systems
Linear Independence
Introduction to Linear Transformations
The Matrix of a Linear Transformation
Linear Models in Business, Science, and Engineering

Matrix Algebra

Matrix Operations
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Characterizations of Invertible Matrices
Partitioned Matrices
Matrix Factorizations
The Leontief Input—Output Model
Applications to Computer Graphics
Subspaces of R^n
Dimension and Rank

Vector Spaces

Vector Spaces and Subspaces
Null Spaces, Column Spaces, and Linear Transformations
Linearly Independent Sets; Bases
Coordinate Systems
The Dimension of a Vector Space
Rank; Change of Basis
Applications to Markov Chains

Eigenvalues and Eigenvectors

Eigenvectors and Eigenvalues
The Characteristic Equation
Diagonalization
Eigenvectors and Linear Transformations

Determinants

Introduction to Determinants
Properties of Determinants

Finite-State Markov Chains

Google and Markov Chains
The Steady-State Vector and Google's PageRank

Orthogonality and Least Squares

Inner Product, Length, and Orthogonality
Orthogonal Sets
Orthogonal Projections
The Gram—Schmidt Process
Least-Squares Problems

MATH 200C: LINEAR ALGEBRA
TENTATIVE SCHEDULE – SPRING 2026

Week Of:	Monday	Tuesday	Wednesday	Thursday	Friday
February 9	*FIRST CLASS*	Assignment 0	Assignment 0		Assignment 1
February 16	Assignment 2		Assignment 3		Assignment 4
February 23	Assignment 5		Assignment 6		Assignment 7
March 2	Assignment 8		**EXAM 1** Assignment 9		Assignment 10
March 9	Assignment 11		Assignment 12		Assignment 13
March 16	Assignment 14		Assignment 15		Assignment 16
March 23	***MID TERM RECESS***	***RECESS***	***MID TERM RECESS***	***RECESS***	***MID TERM RECESS***
March 30	Assignment 17		Assignment 18		Assignment 19
April 6	Assignment 20		**EXAM 2**		***Student Symposium***
April 13	Assignment 21		Assignment 22		Assignment 23
April 20	Assignment 24		Assignment 25		Assignment 26
April 27	Assignment 27		Assignment 28		Assignment 29
May 4	**EXAM 3**		Assignment 30		Assignment 31
May 11	Assignment 32			***Final Exam***	

The College's Scheduling Officer has set Thursday, May 14 from 9 AM to Noon as the date and time for the Final Examination

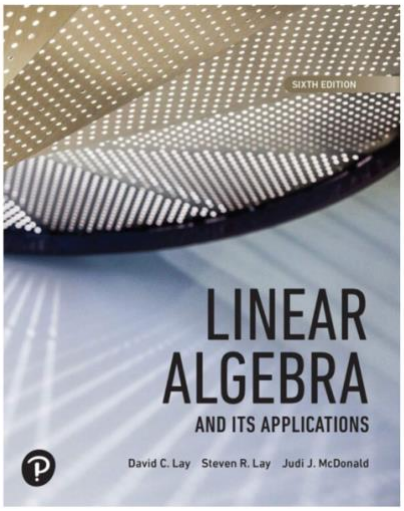
Note: Schedule indicates dates on which assignments are due.

MATH 200C Linear Algebra
Guide To Assignments for Spring 2026

Each assignment is due in class on Monday, Wednesday, and Friday. Homework will generally focus on material introduced in the previous three class meetings. Begin the new assignment as soon after class as practical so that you will have two days to complete it. Review your class notes and read the assigned sections first, and then work the problems. We encourage you to study together and wrestle with hard problems, but submitted solutions should be your own work. Come to class with well-prepared and specific questions on the assignment.

Experience shows that doing the assignments regularly and carefully is the key to doing well in calculus. You should expect to spend at least two hours on homework for every hour in class, and an average of 8 to 10 hours total each week including your review and careful reading.

Most of the readings and exercises will come from our text: David Lay, Steven Lay and Judi McDonald, *Linear Algebra And Its Applications,* Sixth Edition.



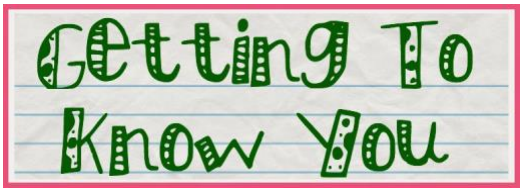
The reading is an important supplement to what goes on in class. It will cover some examples and methods you need and for which you will be held responsible but which we will not have time to cover in class. Read carefully with a pencil and pad next to you. Work out details of calculations you don't understand.

MATH 200C Spring 2026

Assignment 0

(Adapted from *Homework 0* by Alex Lyford)

Due: Wednesday, February 11



Reading

Read carefully "A Note To Students" and Section 1.1 "Linear Equations in Linear Algebra" in our text *Linear Algebra and Its Applications* (Sixth Edition).

Writing

You may submit an electronic copy of this assignment to me (molinick@middlebury.edu) with the subject line: **MATH 200 Assignment 0** or print it out and bring it to Wednesday's class. Make sure you include your name at the top of the document.

Your task is to create a document describing yourself, your goals, and what you hope to get out of our Linear Algebra class. Please provide your name at the top of the first page along with your major or likely major and your anticipated graduate date.

Start with an autobiographical statement about yourself that will help me to get to know a little about you. Where did you grow up? Why did you come to Middlebury? What are your likes and dislikes? Do you have any hobbies that you do regularly? Do you have a major extracurricular activity such as athletics, theatre or *The Campus?*

After the biographical statement tell me about your mathematical, statistical, and computer programming background. What did you like about previous mathematics, statistics, and/or programming classes? What did you dislike? What aspects did you find easy? What aspects did you find challenging?

The next part should discuss your plans for the remainder of your time in college, and what you hope to do after you graduate. Is more schooling the next step, or do you plan to get a job? It's okay to not have any idea what you want to do after graduation, but list some possibilities so that I can better tailor the materials in class to your potential career opportunities.

Finally tell me about your thoughts and expectations for this class. What are you hoping and/or expecting to learn? What do you think the challenges of this course might be? What, if anything, have you heard about this course from your peers? What expectations do you have of me? Feel free to also discuss anything I've failed to ask here!

MATH 200C Spring 2026

Assignments 1 – 6

Assignment 1	Read Section 1.1	§1.1: 1 , 5, 7, 11, 13
Assignment 2	Read Section 1.2	§1.1: 15, 19, 21 §1.2: 1, 3, 7, 11
Assignment 3	Read Section 1.3	§ 1.1: 23, 43, 44 § 1.2: 13,19, 21 § 1.3: 1, 5, 7, 9
Assignment 4	Read Section 1.4	§ 1.2: 23, 41, 44 § 1.3: 13, 19, 21 § 1.4: 1, 2, 3, 5
Assignment 5	Read Section 1.5	§ 1.3: 35, 36 § 1.4: 11, 15, 18 § 1.5: 1, 2, 3, 5
Assignment 6	Read Section 1.6	§ 1.4: 21, 22, 37 § 1.5: 8, 11, 19, 23 § 1.6: 1, 3, 7, 11

ON PROBLEM SOLVING

A major part of your time in Calculus and other courses is devoted to solving problems. It is worth your while to develop sound techniques. Here are a few suggestions.

Think. Before plunging into a problem, take a moment to think. Read the problem again. Think about it. What are its essential features? Have you seen a problem like it before? What techniques are needed?

Try to make a rough estimate of the answer. It will help you understand the problem and will serve as a check against unreasonable answers. A car will not go 1,000 miles in 3 hours; a weight dropped from 10,000 feet will not hit the earth at 5 mph; the volume of a tank is not -275 gal.

Examine the data. Be sure you understand what is given. Translate the data into mathematical language. Whenever possible, make a clear diagram and label it accurately. Place axes to simplify computations. If you get stuck, check that you are using all the data.

Avoid sloppiness.

(a) *Avoid sloppiness in language.* Mathematics is written in English sentences. A typical mathematical sentence is "$y = 4x + 1$." The equal sign is the verb in this sentence; it means "equals" or "is equal to." The equal sign is not to be used in place of "and", nor as a punctuation mark.

Quantities on opposite sides of an equal sign must be equal.

Use short simple sentences. Avoid pronouns such as "it" and "which". Give names and use them. Consider the following example.

"To find the minimum of it, differentiate it and set it equal to zero, then solve it which if you substitute it, it is the minimum."

Better: "To find the minimum of $f(x)$, set its derivative $f'(x)$ equal to zero. Let x_0 be the solution of the resulting equation. Then $f(x_0)$ is the minimum value of $f(x)$."

(b) *Avoid sloppiness in computation.* Do calculations in a sequence of neat, orderly steps. Include all steps except utterly trivial ones. This will help eliminate errors, or at least make errors easier to find. Check any numbers used; be sure that you have not dropped a minus sign or transposed digits.

(c) *Avoid sloppiness in units.* If you start out measuring in feet, all lengths must be in feet, all areas in square feet, and all volumes in cubic feet. Do not mix feet and acres, seconds and years.

(d) *Avoid sloppiness in the answer.* Be sure to answer the question that is asked. If the problem asks for the maximum value of $f(x)$, the answer is not the point where the maximum occurs. If the problem asks for a formula, the answer is not a number.

EXAMPLE Find the minimum of $f(x) = x^2 - 2x + 1$.

Solution 1:

$$2x - 2$$
$$x = 1$$
$$1^2 - 2 \cdot 1 + 1$$
$$0$$

Unbearable. This is just a collection of marks on the paper. There is absolutely no indication of what these marks mean or of what they have to do with the problem. When you write, it is your responsibility to inform the reader what you are doing. Assume he is intelligent, but not a mind reader.

Solution 2:

$$\frac{df}{dx} = 2x - 2 = 0 = 2x = 2 = x = 1$$

$$= f(x) = 1^2 - 2 \cdot 1 + 1 = 0.$$

Poor. The equal sign is badly mauled. This solution contains such enlightening statements as "0 = 2 = 1," and it does not explain what the writer is doing.

Solution 3:

$$\frac{df}{dx} = 2x - 2 = 0, \quad 2x = 2, \quad x = 1.$$

This is better than Solution 2, but contains two errors. Error 1: The first statement, "$\frac{df}{dx} = 2x - 2 = 0$," muddles two separate steps. First the derivative is computed, then the derivative is equated to zero. Error 2: The solution is incomplete because it does not give what the problem asks for, the minimum value of *f*. Instead, it gives the point *x* at which the minimum is assumed.

Solution 4: The derivative of *f* is

$$f' = 2x - 2.$$

At a minimum, **f** ' = 0. Hence

$$2x - 2 = 0, \quad x = 1.$$

The corresponding value of *f* is

$$f(1) = 1^2 - 2 \cdot 1 + 1 = 0.$$

If $x > 1$, then $f'(x) = 2(x-1) > 0$, so *f* is increasing. If $x < 1$, then f '(x) = 2(x-1) < 0, so *f* is decreasing. Hence f is minimal at x = 1, and the minimum values of f is 0.

This solution is absolutely correct, but long. For homework assignments the following is satisfactory (check with your instructor):

Solution 5:

$$f'(x) = 2x - 2.$$

At min, $f' = 0$, $2x - 2 = 0$, $x = 1$. For $x > 1$, $f'(x) = 2(x-1) > 0$, f□: for $x < 1$, $f'(x) = 2(x-1) < 0$, f□.

Hence $x = 1$ yields min,

$$f_{min} = f(1) = 1^2 - 2 \cdot 1 + 1 = 0.$$

The next solution was submitted by a student who took a moment to think.

Solution 6:

$$f(x) = x^2 - 2x + 1 = (x-1)^2 \geq 0.$$

But

$$f(1) = (1-1)^2 = 0.$$

Hence the minimum value of $f(x)$ is 0.

- from *A First Course in Calculus*
by Flanders, Korfhage and Price

ON STUDYING AND LEARNING MATHEMATICS

Past students have found that some ways are far more effective than others in studying and learning mathematics. Here are some suggestions and pointers that may help you in budgeting the time you can devote to mathematics, preparing for examinations, and learning and understanding the material in a way that promotes long-range retention:

1. Do all reading assignments actively. Keep a pencil and scratch paper at hand. Mark up the pages of the book. Write in any questions you may have. Verify examples given by writing out the details yourself.

2. Plan to do all reading assignments several times. In mathematics courses, reading assignments are seldom more than a few pages long. They often contain, however, subtle ideas which require repeated study before they are mastered. You should read the appropriate section of the text before the class in which it will be discussed, read it again before beginning the homework assignment, and read it a third time after you have completed the homework.

3. Follow the advice in (1) above when reviewing your lecture notes. You should try to go over your lecture notes as soon as possible after the class session has ended. Definitely review the notes before attempting the homework.

4. Do all homework sets on time. Don't let yourself fall behind. If you have difficulty with a problem, especially one that is more theoretical, do the following:

   (a) Write out the relevant definitions and results. It may now be a small step to complete the problem.

   (b) Ask whether you can think of a simpler but related problem, and tackle that one first. Is there a special case of the general result? Do you know how to solve the problem in this special case? This approach usually provides insight for attacking the original problem.

5. Do not spend hours sitting still, thinking, reading, studying and reviewing problem solutions! While these approaches may be helpful for other courses and some time should be spent on these activities in mathematics courses, there are more productive paths to learning in mathematics. Spend your time writing out solutions to new problems, deriving relationships, writing down clear definitions, and outlining the steps of a proof. These activities provide a better way to prepare for an examination.

6. Pay a great deal of attention to definitions. Write them out yourself and think about them. Write out examples that do and do not satisfy the definitions. Ask yourself how the definition says something different from its intended meaning if the order of the words is shifted.

7. Begin reviewing for examinations a week early. Use small chunks of time. Tackle those topics you have found difficult; with hindsight they are often easier. Do NOT plan on spending a whole day of study just before an exam. This is almost always an inefficient way to budget your time.

8. Review solutions for homework problems as soon as you get them, and write up (for your own enlightenment) those problems which caused you difficulty.

9. Write down questions that arise as you go along. Bring them with you to class, to review sessions, and to your instructor's office hours.

MATH 200C: Linear Algebra

Spring 206

What To Do By Tomorrow

1) Read through the documents in this packet.

2) Access the course website http://s26.middlebury.edu/MATH0200C and check out some of the links.

3) Obtain a copy of our text. Familiarize yourself with the structure of the book. Read carefully Section 1 of Chapter 1.

4) Purchase a loose leaf binder to store the various handouts about the course that will be distributed, your class notes and the homework. You will accumulate a large number of loose sheets of paper from this course during the term; it's very helpful to keep them organized. You may also wish to obtain some graph paper.

5) Begin work on Assignment 0, the assignment due to be turned in on Wednesday.

6) **Don't hesitate to come in to see me if you need some help or have questions.**