

Highlights from §1.7: Linear Independence

Definition: An indexed set of vectors $\{ \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p \}$ in \mathbb{R}^n is **linearly independent** if the vector equation

$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \dots + x_p\mathbf{v}_p = \mathbf{0}$$

has *only* the trivial solution.

The set $\{ \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p \}$ is **linearly dependent** if there exist weights c_1, c_2, \dots, c_p **not all zero** such that

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \dots + c_p\mathbf{v}_p = \mathbf{0}$$

Observations

The columns of a matrix A are linearly independent if and only if the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

A set of exactly one vector is linearly independent if and only if the vector is not the zero vector.

A set of exactly two vectors is linearly independent if and only if neither vector is a nonzero scalar multiple of the other vector.

Theorem 7 (Characterization of Linearly Dependent Sets): An indexed set of two or more vectors $\{ \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p \}$ in \mathbb{R}^n is **linearly dependent** if at least one of the vectors is a linear combination of the other vectors.

Theorem 8: If a set contains more vectors than there are entries in each vector, then the set is linearly dependent; any set of vectors $\{ \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p \}$ in \mathbb{R}^n is linearly dependent if $p > n$.

If $p \leq n$, there is no guarantee that the set is linearly independent

Theorem 9: If a set of vectors contains the zero vector, then the set is linearly dependent.

There are sets of vectors, all nonzero, which are linearly dependent.

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