Vector Spaces

Change of Basis

Theorem 15: Let $\mathcal{B} = \{\mathbf{b_1}, \ldots, \mathbf{b_n}\}$ and $C = \{\mathbf{c_1}, \ldots, \mathbf{c_n}\}$ be bases for the same vector space V.

There exists a unique $n \times n$ matrix $\underset{C \leftarrow \mathcal{B}}{P}$ such that

$$[\mathbf{x}]_C = \underset{C \leftarrow \mathcal{B}}{P} [\mathbf{x}]_B$$

The columns of $\underset{C \leftarrow \mathcal{B}}{P}$ are the C -coordinates vectors of the vectors in the basis $\mathcal{B}$; that is,

$$\underset{C \leftarrow \mathcal{B}}{P} = [\ [\boldsymbol{b_1}]_C \quad [\boldsymbol{b_2}]_C \quad \ldots . [\boldsymbol{b_n}]_C\ ]$$

The matrix $\underset{C \leftarrow \mathcal{B}}{P}$ is called the **change-of-coordinates matrix from $\mathcal{B}$ to C.**