MATH 200C Linear Algebra

Vector Spaces

Vector Spaces and Subspaces

Definition: A **vector space** is a nonempty set V of objects, called vectors, on which are defined two operations, called addition and multiplication by scalars (real numbers), subject to the ten axioms (or rules) listed below. The axioms must hold for all vectors **u**, **v**, and **w** in V and for all scalars c and d.

1. The sum of **u** and **v**, denoted by **u** + **v**, is in V.	6. The scalar multiple of **u** by c, denoted by c**u**, is in V
2. **u** + **v** = **v** + **u**.	7. c(**u** + **v**) = c**u** + c**v**
3. (**u** + **v**) + **w** = **u** + (**u** + **w**)	8. ($c + d$) /**u** = c**u** + d**u**
4. There is a zero vector **0** in V such that **u** + **0** = **u**	9. c(d**u**) = (cd)**u**.
5. For each **u** in V, there is a vector -**u** in V such that **u** + -**u** = **0**	10. 1**u** = **u**

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Simple Consequences: For each **u** in V and scalar c, 0**u** = **0**, c**0** = **0**, and -**u** = (-1)**u**

Subspaces

In many problems, a vector space consists of an appropriate subset of vectors from some larger vector space. In this case, only three of the ten vector space axioms need to be checked; the rest are automatically satisfied.

Definition: A **subspace** of a vector space V is a subset H of V that has three properties:

a. The zero vector of V is in H

b. H is closed under vector addition. That is, for each **u** and **v** in H , the sum **u** + **v** is in H

c. H is closed under multiplication by scalars. That is, for each **u** in H and each scalar c, the vector c**u** is in H .

Theorem 1: If $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ are in a vector space V, then span ($\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$) is a subspace of V.

Examples of Vector Spaces

- $\mathbb{R}^n$, for $n \geq 1$
- All arrows in 3 dimensional space
- Doubly infinite sequences of numbers $\{\ldots\ y_{-2}, y_{-1}, y_0, y_1, y_2, y_3, \ldots\}$
- P_n, polynomials of degree $\leq n$
- Real-Values functions on an interval I.