The Characteristic Equation

Definition: An **eigenvector** of an $n \times n$ matrix A is a nonzero vector **x** such that A**x** = λ**x** for some scalar λ. A scalar λ is called an **eigenvalue** of A if there is a nontrivial solution **x** of A**x** = λ**x**; such an **x** is called an **eigenvector corresponding to λ**

A scalar λ is an eigenvalue of a square matrix A if and only if λ satisfies the characteristic equation $\det(A - \lambda I) = 0$.

If A is an n by n matrix, the $\det(A - \lambda I)$ is a polynomial in λ of degree n called the **Characteristic Polynomial** of A.

The **Characteristic Equation** is the polynomial equation $\det(A - \lambda I) = 0$.

The eigenvalues of A are solutions of this equation (= the roots of the polynomial)

Example: $A = \begin{bmatrix} -17 & -30 \\ 10 & 18 \end{bmatrix}$ has characteristic polynomial

$$\lambda^2 - \lambda - 6 = (\lambda - 3)(\lambda + 2)$$

So eigenvalues are 3 and -2.

Definition: An eigenvalue r has **algebraic multiplicity *k*** if k is the largest integer such that $(\lambda - r)k$ is a factor of the characteristic polynomial.

Definition: Two n by n square matrices A and B are **similar** if there is an n by n invertible matrix P such that $P^{-1}A\,P = B$.

Theorem: If n by n square matrices A and B are similar, then they have the same characteristic polynomials and thus the same eigenvalues with the same multiplicities.