Eigenvalues and Eigenvectors

<u>Definition</u>: An **eigenvector** of an $n \times n$ matrix A is a nonzero vector $\mathbf{x}$ such that $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ. A scalar λ is called an **eigenvalue** of A if there is a nontrivial solution $\mathbf{x}$ of $A\mathbf{x} = \lambda\mathbf{x}$; such an $\mathbf{x}$ is called an **eigenvector corresponding to λ**

Warning: Although we can use row reduction to find eigenvectors, it cannot be used to find eigenvalues. An echelon form of a matrix A usually does not display the eigenvalues of A.

Theorem 1: The eigenvalues of a triangular matrix are the entries on its main diagonal.

Theorem 2: If $\mathbf{v}_1, \ldots, \mathbf{v}_r$ are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \ldots, \lambda_r$ of an $n \times n$ matrix A, then the set $\{ \mathbf{v}_1, \ldots, \mathbf{v}_r \}$ is linearly independent.

Theorem: (Extending the Invertible Matrix Theorem): An $n \times n$ matrix A is invertible if and only if the number 0 is not an eigenvalue of A.

A scalar λ is an eigenvalue of a square matrix A if and only if λ satisfies the characteristic equation $\det(A - \lambda I) = 0$.