MATH 200C Spring 2026

Project 1: Naïve Matrix Multiplication

Due: Friday, March 20

Introduction: We **defined** scalar multiplication of a matrix and addition of two matrices in a naïve way: just carry out the indicated operations on each corresponding entry; that is,

The *ij*th entry of (*rA*) is the scalar *r* multiplied by the *ij*th entry of *A*

The *ij*th entry of (*A* + *B*) is the sum of the *ij*th entry of *A* and the *ij*th entry of *B*.

With these definitions, we obtain a treasure trove of correct, useful properties. Theorem 1 of Section 2.1 in our text summarizes half a dozen of these.

We then proceeded to define a way to multiply together certain pairs of matrices. If *A* is an $m \times n$ matrix and B is an $n \times p$ matrix, then AB is an $m \times p$ matrix. [See Definition and justification on page 101.]. This definition yielded a number of nice properties as summarized in Theorem 2 of Section 2.1.

This definition may have seemed a bit odd. A more “natural” approach may have been to emulate the ideas behind scalar multiplication and matrix addition and use the product of the corresponding entries. The aim of this project is to explore this alternate definition, which I will call *naïve multiplication.*

Definition: The **naïve product** of two matrices is the matrix denoted **A # B** such that

The *ij*th entry of *A* # *B* is the product of the *ij*th entry of *A* and the *ij*th entry of *B*.

Example: If $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}$

then $A\#B = \begin{bmatrix} (1)(4) & (2)(-3) \\ (3)(3) & (-4)(-5) \end{bmatrix} = \begin{bmatrix} 4 & -6 \\ 9 & 20 \end{bmatrix}$

Note that the naïve product of two matrices is only defined if the matrices have the same size. (They do not have to be square matrices. The naïve product A#B would be defined, for example, if *A* and *B* were both 2 × 3 matrices.)

Part I

Part I is designed to give you needed practice in developing and writing short proofs of mathematical assertions. One of the goals of 200 level courses is to starting building such skills. Your proofs in Part I should all be your original work, created on your own without reference to external sources. You may find it helpful to emulate proofs presented in class or in our textbook.

You may make use of the standard properties of arithmetic for real numbers; for example, $ab = ba$ for all real numbers a and b. Your proofs do not have to be typed, but they must be legibly written.

1. If A is an $m \times n$ matrix, find an $m \times n$ matrix J so that $A\#J = J\#A = A$. Such a matrix J is called the **identity matrix** for naïve multiplication.
2. In contrast to ordinary matrix multiplication, prove that naïve multiplication is commutative; that is $A\#B = B\#A$ whenever A and B have the same size.
3. Formulate analogs to the 5 properties of ordinary matrix multiplication for naïve multiplication that are stated in Theorem 2 of Section 2.1. For example, the analog to (c) would read $(B + C) \# A = (B \# A) + (C \# A)$.
   For each of these 5 assertions, provide a complete, clear, and convincing proof if the assertion is true or provide a counterexample if you believe it is false.
4. Find two nonzero matrices A and B such that $A\#B = 0$, the zero matrix.
5. If $A\#C = B\#C$, can we conclude that $A = B$?
6. Keeping the same definition of transpose, is the assertion in part (d) of Theorem 3 true for naïve multiplication? That is, is $(A\#B)^T = B^T \# A^T$? Provide a proof or counterexample.
7. How would you define a naïve inverse for a matrix A? Under what conditions on its entries does A have a naïve inverse? Is the naïve inverse unique?
8. Describe a simple algorithm for computing the naïve inverse of a matrix A.
9. Which analogs to the properties listed in Theorem 6 of Section 2.2 are true for naïve inverses? In particular,
   (a) Is the naïve inverse of the (naïve inverse of A) always equal to A?
   (b) Is the naïve inverse of $A\#B$ equal to product of the naïve inverses of A and B in reverse order?
   (c) Is the naïve inverse of A^T equal to the transpose of the naïve inverse of A?
10. Suggest a notation for the naïve inverse of A, avoiding confusion with the notation A^{-1}

Part II

In Part II, you are free to consult external references such as the Internet. The goal is t find an interesting real-world application of naïve matrix multiplication (there are many!) and write a clear exposition of the application that will be understandable to your classmates. You will likely need 3 to 5 pages to present an interesting account.