MATH 200C Notes on Exam 1

1. Suppose the matrix $\begin{bmatrix} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 4 & 1 & 2 \end{bmatrix}$ is the augmented matrix of a linear system.

State in words the next two elementary row operations that should be performed in the process of solving the system. *Multiply Row 2 by ½. Then Add 6 times Row 2 to Row 1.*

2. David "Give No Quarter" Dorman hates those 25 cent coins. He only carries pennies (1 cent coins), Nickels (5 cent coins) and dimes (10 cent coins). One morning he discovers that there are 13 coins in his pocket whose total value is 83 cents.

(a) Set up a system of linear equations whose solution will tell us how many of each coin David has. *Let P, N and D represent the number of pennies, nickels and dimes, respectively. Then the equations are:*

$$1P + 1N + 1D = 13$$
$$1P + 5N + 10D = 83$$

(b) Solve the system by the Gauss-Jordan procedure. Reducing the augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 13 \\ 1 & 5 & 10 & 83 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 1 & 13 \\ 0 & 4 & 9 & 70 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 1 & 13 \\ 0 & 1 & 9/4 & 70/4 \end{array}\right] \to \left[\begin{array}{ccc|c} 0 & 1 & -5/4 & -18/4 \\ 0 & 1 & 9/4 & 70/4 \end{array}\right]$$

So the solution is P = 5/4 D – 18/4, N = 70/4 -9/4 D, D is free.

(c) Use your solution in (b) to determine how many of each coin Mr. Dorman has. Since P, N, and D must be nonnegative whole numbers, we must have 5D≥ 18 (which makes D ≤4) and 70≥9D (which makes D ≤ 7). The possible values for D are 4, 5, 6, 7 but only D = 6 gives integers for P (3) and N (4). Mr. Dorman has 3 pennies, 4 nickels, and 6 dimes.

3. For what values of a and b is the following system consistent?

$$2x - 1y = a$$
$$-6x + 3y = b$$

$$\left[\begin{array}{cc|c} 2 & -1 & a \\ -6 & 3 & b \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -1/2 & a/2 \\ -6 & 3 & b \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -1/2 & a/2 \\ 0 & 0 & 3a+b \end{array}\right].$$

The system is consistent exactly when $3a + b = 0$.

4. Barack Obama was born on August 4, 1961 and Mitt Romney was born on March 12, 1947. In their honor, Let A = $\begin{bmatrix} 8 & 4 \\ 6 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 12 \\ 4 & 7 \end{bmatrix}$

(a) Compute the entries in row 1, column 2 for each of AB and BA

$$(AB)_{1,2} = [\ 8 \quad 4\ ]\begin{bmatrix} 12 \\ 7 \end{bmatrix} = 8\times 12 + 4\times 7 = 124 \text{ but } (BA)_{1,2} = [\ 3 \quad 12\ ]\begin{bmatrix} 4 \\ 1 \end{bmatrix} = 3\times 4 + 12\times 1 = 24$$

(b) Is $AB = BA$? Explain. No, since $(AB)_{1,2} \neq (BA)_{1,2}$ (c) Transpose of B. $\begin{bmatrix} 3 & 4 \\ 12 & 7 \end{bmatrix}$

5. Suppose **u** and **v** are two vectors in R^{2012}.

(a) Show that the set {**u**,**v**} is a linearly dependent set if **v** is a scalar multiple of **u.**

If **v** = *c***u,** then *c***u** -1**v** = **0** is an "interesting" linear combination of **u** and **v** that equals **0**. By definition **{u,v}** is a linearly dependent set.

(b) Is the converse true? That is, if {**u**,**v**} is a linearly dependent set, then must **u** be a scalar multiple of **v**? If {**u**,**v**} is a linearly dependent set, then there are scalars a and b, not both zero, such that a**u** + b**v** = 0 so a**u** = -b**v**. If $a \neq 0$, then **u** = (-b/a)**v,** while if $b \neq 0$, then **v** = (-a/b)**u**, so at least one of the vectors is a scalar multiple of the other. If **v** is the zero vector, then we can let $b = 1$, $a = 0$ and then choose any vector **u**. Thus **u** need not be a scalar multiple of **v.** If **v** is a nonzero vector, then **u** must be a scalar multiple of **v** since we could not have a =0.

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6. Let T be a linear transformation from R^n to R^m.

(a) Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ be vectors in R^n and suppose { $T(\mathbf{v}_1)$, $T(\mathbf{v}_2)$, $T(\mathbf{v}_3)$, $T(\mathbf{v}_4)$ } is a linearly independent set. Show that { $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$} must be a linearly independent set.

Suppose, to the contrary, that { $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$} is a linearly dependent set. Then there are constants a,b,c, and d, not all 0 such that a **v1** + b $\mathbf{v}_2$ + c $\mathbf{v}_3$ + d $\mathbf{v}_4$= **0**. Then, by properties of linear transformations,

0 = T(**0**) = T(a **v1** + b $\mathbf{v}_2$ + c $\mathbf{v}_3$ + d $\mathbf{v}_4$**)** = a $T(\mathbf{v}_1)$ + b $T(\mathbf{v}_2)$ + c $T(\mathbf{v}_3)$ + d $T(\mathbf{v}_4)$ is an "interesting" linear combination of $T(\mathbf{v}_1)$, $T(\mathbf{v}_2)$, $T(\mathbf{v}_3)$, $T(\mathbf{v}_4)$ equal to **0** so{ $T(\mathbf{v}_1)$, $T(\mathbf{v}_2)$, $T(\mathbf{v}_3)$, $T(\mathbf{v}_4)$ } would be a linearly dependent set which contradicts the assumption that it is linearly independent.

(b) Suppose $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4$ are vectors in R^n and { $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4$} is a linearly independent set. Must it always be true that {$T(\mathbf{u}_1)$, $T(\mathbf{u}_2)$, $T(\mathbf{u}_3)$, $T(\mathbf{u}_4)$ } is a linearly independent set? **NO**. Let T be the transformation that sends every vector to **0.**

7. The reduced row echelon form for the augmented matrix of a certain linear system of 3 equations in 5 unknowns has the form $\begin{bmatrix} 1 & 0 & -2 & 33 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$. Write down the general solution.

$x_1 = -24 + 2x_3 - 33x_4$

$x_2 = -7 + 2x_3 - 2x_4$

x_3, x_4 are free

$x_5 = 4$

8. Alka Seltzer contains sodium bicarbonate ($NaHCO_3$) and citric acid ($H_3C_6H_5O_7$). When a tablet is dissolved in water, it plops and fixes, producing sodium citrate, water and carbon dioxide. The unbalanced reaction looks like

$$NaHCO3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7\ H_2O + CO_2$$

(a) Write down a system of linear equations whose solution would help you balance the reaction. You do NOT have to solve the system.

Let x1 be the coefficient of NaHC03, x2 the coefficient of $H_3C_6H_5O_7$, x3 the coefficient of $Na_3C_6H_5O_7$, x4 the coefficient of H_2O and x5 the coefficient of $C0_2$. The following vectors list the numbers of atoms of sodium(Na), hydrogen(H), carbon(C) and oxygen(O)

$$NaHCO_3: \begin{bmatrix}1\\1\\1\\3\end{bmatrix},\ H_3C_6H_5O_7: \begin{bmatrix}0\\8\\6\\7\end{bmatrix},\ Na_3C_6H_5O_7: \begin{bmatrix}3\\5\\6\\7\end{bmatrix},\ H_2O: \begin{bmatrix}0\\2\\0\\1\end{bmatrix},\ CO_2: \begin{bmatrix}0\\0\\1\\2\end{bmatrix} \begin{matrix}\text{sodium}\\\text{hydrogen}\\\text{carbon}\\\text{oxygen}\end{matrix}$$

The coefficients satisfy the vector equation

$$x_1\begin{bmatrix}1\\1\\1\\3\end{bmatrix} + x_2\begin{bmatrix}0\\8\\6\\7\end{bmatrix} = x_3\begin{bmatrix}3\\5\\6\\7\end{bmatrix} + x_4\begin{bmatrix}0\\2\\0\\1\end{bmatrix} + x_5\begin{bmatrix}0\\0\\1\\2\end{bmatrix} \begin{matrix}\textit{sodium}\\\textit{hydrogen}\\\textit{carbon}\\\textit{oxygen}\end{matrix}$$

(b) Display the augmented matrix for this system of equations.

$$\left[\begin{array}{ccccc|c} 1 & 0 & -3 & 0 & 0 & 0\\ 1 & 8 & -5 & -2 & 0 & 0\\ 1 & 6 & -6 & 0 & -1 & 0\\ 3 & 7 & -7 & -1 & -2 & 0 \end{array}\right]$$