MATH 200C Sample Examination 1 Spring 2014

Please do all of the problems on this examination. ***You must show all your work to receive full or partial credit. Correct answers with little or now intermediate work shown will receive minimal credit.***
Please write legibly, Use careful and complete sentences to provide requested explanations. When numerical answers have fractional form, please leave them as fractions; do not convert to decimals.

1. Suppose the matrix $\left[\begin{array}{cccc|c} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 4 & 1 & 2 \end{array}\right]$ is the augmented matrix of a linear system. State in words the next two elementary row operations that should be performed in the process of solving the system.

2. David "Give No Quarter" Dorman hates those 25 cent coins. He only carries pennies (1 cent coins), Nickels (5 cent coins) and dimes (10 cent coins). One morning he discovers that there are 13 coins in his pocket whose total value is 83 cents.

(a) Set up a system of linear equations whose solution will tell us how many of each coin David has.

(b) Solve the system by the Gauss-Jordan procedure.

(c) Use your solution in (b) to determine how many of each coin Mr. Dorman has.

3. For what values of a and b is the following system consistent?

$$2x - 1y = a$$
$$-6x+3y = b$$

4. Barack Obama was born on August 4, 1961 and Mitt Romney was born on March 12, 1947. In their honor, Let A = $\left[\begin{array}{cc} 8 & 4 \\ 6 & 1 \end{array}\right]$ and B = $\left[\begin{array}{cc} 3 & 12 \\ 4 & 7 \end{array}\right]$

(a) Compute the entries in row 1, column 2 for each of AB and BA
(b) Is $AB = BA$? Explain. (c) Display the transpose of B.

5. Suppose **u** and **v** are two vectors in R^{2012}.

(a) Show that the set {**u**,**v**} is a linearly dependent set if **v** is a scalar multiple of **u.**

(b) Is the converse true? That is, if {**u**,**v**} is a linear dependent set, then must **u** be a scalar multiple of **v**? Provide a proof if you think it must be true or a counterexample if you believe it may be false.

6. Let T be a linear transformation from R^n to R^m.

(a) Let $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}$ be vectors in R^n and suppose { $T(\mathbf{v_1})$, $T(\mathbf{v_2})$, $T(\mathbf{v_3})$, $T(\mathbf{v_4})$ } is a linearly independent set. Show that { $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}$} must be a linearly independent set.

(b) Suppose $\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}$ are vectors in R^n and { $\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}$} is a linearly independent set. Must it always be true that {$T(\mathbf{u_1})$, $T(\mathbf{u_2})$, $T(\mathbf{u_3})$, $T(\mathbf{u_4})$ } is a linearly independent set? Prove that it must be linearly independent or give an example when it is not.

7. The reduced row echelon form for the augmented matrix of a certain linear system of 3 equations in 5 unknowns has the form $\left[\begin{array}{ccccc|c} 1 & 0 & -2 & 33 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$. Write down the general solution.

8. Alka Seltzer contains sodium bicarbonate ($NaHCO_3$) and citric acid ($H_3C_6H_5O_7$). When a tablet is dissolved in water, it plops and fizzes, producing sodium citrate, water and carbon dioxide. The unbalanced reaction looks like

$$NaHCO3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + H_2O + CO_2$$

(a) Write down a system of linear equations whose solution would help you balance the reaction. You do NOT have to solve the system.

(b) Display the augmented matrix for this system of equations.