MATH 200C Sample Examination 2

1. Let R^4 be the usual Euclidean 4-dimensional space.
(a) List the elements of a 5 element subset of R^4 which spans R^4 but is not linearly independent
(b) List the members of a 3 element subset of R^4 which is linearly independent but does not span R^4.
(c) List the members of a nonempty subset of R^4 which is neither linearly independent nor spanning.
(d) List the members of a subset of R^4 which is a basis for R^4.

2. Short answer Questions:
(a) Suppose A is a 9 x 9 matrix such that $A^3 = A$. What are the possible values for the determinant of A?
(b) Suppose A and B are invertible 11 x 11 matrices. What is the inverse of ABA^{-1}?
(c) Give an example of a vector space which does **not** have a finite basis.
(d) What is the dimension of the vector space of all 3×2 matrices?
(e) If A is a 7×9 matrix with a two dimensional null space, what is the rank of A?
(f) Can a 6×9 matrix have a two-dimensional null space? Explain.
(g) What is the dimension of the vector space of all polynomials of degree ≤ 12?

3. A personalized question. Let b_1 = the number of the month in which you were born (1 = January, 2 = February, etc..), b_2 = the day of the month on which you were born, and b_3 = the year of your birth. Form the vector $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

Suppose A is any 3×3 matrix such that the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution. Explain why A must be invertible. [Note: no calculations are required]

4. Let $S = \{\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}\}$ be a linearly independent subset of a vector space V and suppose $\mathbf{w}$ is a member of V which is not in the span of S. Show that $\{\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}, \mathbf{w}\}$ is a linearly independent set.

5. Let T be a linear transformation from R^{23} to R^{23}
(a) Show that the set of vectors $\mathbf{X}$ in R^2 such that $T(\mathbf{X}) = 3\mathbf{X}$ is a subspace of R^2.
(b) If T is the particular matrix transformation from R^2 to R^2 for which $T(\mathbf{e_1}) = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$ and $T(\mathbf{e_2}) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ show that $T(\mathbf{X}) = 3\mathbf{X}$ if $\mathbf{X} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

6. The reduced row echelon form of $A = \begin{bmatrix} 7 & 21 & 4 & 112 & 11 \\ 12 & 37 & 3 & 170 & -16 \\ -4 & -2 & -2 & -62 & -4 \\ 16 & 48 & 12 & 276 & 48 \end{bmatrix}$ is $R = \begin{bmatrix} 1 & 0 & 0 & -3 & 9 \\ 0 & 1 & 0 & 5 & -4 \\ 0 & 0 & 1 & 7 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Find each of the following:

(a) Number of pivot columns of A
(b) Number of pivot positions in R.
(c) rank of A
(d) Dimension of the column space of A
(e) nullity of A (dimension of null space of A)
(f) nullity of the transpose of A
(g) a basis for the row space of A
(h) a basis for the column space of A
(i) Describe a procedure (you do not have to carry out the computations) for finding a basis for the row space which is made up of some of the rows of A.

7. Let S be the parallelogram determined by the vectors $\mathbf{b}_1 = [4, -7]^T$ and $\mathbf{b}_2 = [0,1]^T$ and let A be the matrix $\begin{bmatrix} 7 & 2 \\ 1 & 1 \end{bmatrix}$. Compute the area of the image of S under the linear transformation whose matrix representation is A.

8. (Leontief Input-Output Model). Consider the production model $\mathbf{x} = C\mathbf{x} + \mathbf{d}$ for an economy with two sectors where $C = \begin{bmatrix} .2 & .5 \\ .6 & .1 \end{bmatrix}, \mathbf{d} = \begin{bmatrix} 112 \\ 84 \end{bmatrix}$. Determine the production level vector necessary to satisfy the final demand.