MATH 200C Sample Exam 3

Note: To save space, we've written vectors in $\mathbb{R}^n$ horizontally rather than the usual vertical format; for example $\mathbf{v}_2 = (2,4,6)$ denotes the vector $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

1. *(Short Answer Questions)*
   (a) Define a *vector space*. List all axioms required.
   (b) What is a *subspace*? State the three conditions required to verify that a subset is a subspace.
   (c) Define 2 of these 3 terms: *Linear Independence, Basis, Dimension of a Vector Space*.
   (d) What does it mean for a matrix to be *diagonalizable*?

2. *True / False (justify briefly)*

(a) Every set of 3 vectors in $\mathbb{R}^3$ is linearly independent.
(b) If a set spans a vector space, then it is automatically a basis.
(c) A matrix with 3 distinct eigenvalues is diagonalizable.
(d) The zero vector can be an eigenvector.
(e) Similar matrices have the same eigenvalues.

3. *Computations*

(a) Determine whether the set $W = \{(x, y, z) \in \mathbb{R}^3 : x + 2y - z = 0\}$ is a subspace of $\mathbb{R}^3$.
(b) Determine whether the set of vectors
$\mathbf{v}_1 = (1,2,3), \mathbf{v}_2 = (2,4,6), \mathbf{v}_3 = (1,0,1)$ is linearly independent.
(c) Find a basis and the dimension for the span of $\{(1,0,1), (2,1,3), (0,1,1)\}$
(d) Let $B = \{(1,1), (1,-1)\}$ Find the coordinate vector of $(3,1)$ relative to basis B.

4. *Eigenvalues and Eigenvectors*

(a) Find the eigenvalues of $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$; for each eigenvalue, find a corresponding eigenvector.
(b). Determine whether the matrix $A = \begin{bmatrix} 4 & 1 \\ 0 & 4 \end{bmatrix}$ is diagonalizable. Explain.
(c) Diagonalize (if possible): $A = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$

5. *Proofs / Deeper Understanding*

(a). Prove that any two bases of a vector space have the same number of elements.
(b). Show that eigenvectors corresponding to distinct eigenvalues are linearly independent.

6. *Application*

(a). Consider a transformation represented by matrix A. Explain how diagonalization simplifies computing A^k.
(b) A matrix has eigenvalues 2, 2, and 5. What additional information do you need to determine whether it is diagonalizable?
(c) The matrix below shows the likelihood that individuals will switch between and IOS and an Android smartphone. In the long run, what percentage of smartphone operators would you expect have an Android smartphone.

From: *IOS*	From: *Android*	To
.70	.15	*IOS*
.30	.85	*Android*