MATH 200 Linear Algebra Spring 2010 Final Exam

1. Let A = $\begin{bmatrix} 1 & 3 & -2 & 5 & -3 \\ 2 & 7 & -3 & 7 & -5 \\ 3 & 11 & -4 & 10 & -9 \end{bmatrix}$

(a) Find a basis for the null space of A
(b) Find a basis for the column space of A

2. Show that there is a unique linear transformation T from R^2 to R^2 such that $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

3. Show that the set $\{x^3, 3x^2 - 6x, 6x - 12, 6\}$ is a basis for P_3.

4. (a) Show that for any $m \times n$ matrix A and any nonzero vector **b** in R^m, if **x** is a particular solution to $A\mathbf{v} = \mathbf{b}$ and **h** is in the null space of A, then **x + h** is also a solution to $A\mathbf{v} = \mathbf{b}$.

(b) Show that if **x** is a particular solution to $A\mathbf{v} = \mathbf{b}$, then *every* solution to $A\mathbf{v} = \mathbf{b}$ has the form **x + h**, where **h** is is in the null space of A.

(c) Use (b) (even if you haven't proved it) and your work on Problem 1 to give the general solution to $A\mathbf{v} = \mathbf{b}$ where A is the matrix from Problem 1 and **b** = $\begin{bmatrix} 3 \\ 7 \\ 11 \end{bmatrix}$

5. Let $C = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ and V be the space of all 2×2 matrices. Define a function T from V to V by $T(A) = CA$ for all A in V.

(a) Show that T is a linear transformation. Is T an isomorphism? Explain.
(b) Find the matrix for T relative to the standard basis for V.

6. Let W be the plane in R^3 with basis $\left\{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}\right\}$

(a) Find an orthonormal basis for W.
(b) Find the projection of v = $\begin{bmatrix} 12 \\ 8 \\ 16 \end{bmatrix}$ onto W.
(c) Draw a sketch of what you've just done.

7. Choose ONE of the following statements and prove it:
(a) If A and B are similar matrices, then A^2 is similar to B^2.
(b) Let A, C and D be $n \times n$ matrices, C invertible, such that $A = CD$. Then the matrix DC is similar to A.

8. Prove that if $v_1, v_2, \ldots v_k$ are all eigenvectors of a square matrix A corresponding to the same eigenvalue λ, then any nonzero linear combination of these vectors is also an eigenvector of A corresponding to λ.

9. The growth of a bacterial colony is observed at various numbers of hours after Noon on May 7.

Hours	12	23	30	38	47	59
Bacteria (in thousands)	11	21	31	40	49	59

Use linear algebra techniques to find the equation of the least-squares line that best fits this data. How large would you estimate the colony will be at the 70th hour? At what time would you estimate that colony reached 350,000 bacteria?