MATH 200C: Linear Algebra
Some Notes on Assignment 14

§ 2.2: 31; § 2.3: 15, 17, 21; § 2.4: 1, 3

Problems from §2.2

31. Suppose A is invertible. By Theorem 5, the equation $A\mathbf{x} = \mathbf{0}$ has only one solution, namely, the zero solution. This means that the columns of A are linearly independent, by a remark in Section 1.7.

Problems from §2.3: 15, 17, 21

15. False. Statement (g) of the IMT is true only for invertible matrices.

17. True, by the IMT. If the equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then statement (d) of the IMT is false. In this case, all the lettered statements in the IMT are false, including statement (c), which means that A must have fewer than n pivot positions.

21. If a square upper triangular $n\times n$ matrix has nonzero diagonal entries, then because it is already in echelon form, the matrix is row equivalent to I_n and hence is invertible, by the IMT. Conversely, if the matrix is invertible, it has n pivots on the diagonal and hence the diagonal entries are nonzero.

Problems from §2.4: 1, 3

1. Apply the row-column rule as if the matrix entries were numbers, but for each product always write the entry of the left block-matrix on the *left*.

$$\begin{bmatrix} I & 0 \\ E & I \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} IA+0C & IB+0D \\ EA+IC & EB+ID \end{bmatrix} = \begin{bmatrix} A & B \\ EA+C & EB+D \end{bmatrix}$$

3. Apply the row-column rule as if the matrix entries were numbers, but for each product always write the entry of the left block-matrix on the *left*. $\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}\begin{bmatrix} W & X \\ Y & Z \end{bmatrix} = \begin{bmatrix} 0W+IY & 0X+IZ \\ IW+0Y & IX+0Z \end{bmatrix} = \begin{bmatrix} Y & Z \\ W & X \end{bmatrix}$