

MATH 224A : Vector Calculus

Course Description/Syllabus
Spring Term 2026

Course Title: Vector Calculus

Catalog Description: In this course students will apply the concepts and tools of linear algebra to analyze functions of multiple variables where domain and codomains are vectors, possibly of different dimensions. Students will learn introductory vector analysis, partial differentiation, multiple integration, line integrals, elementary vector field theory, and their applications.

Additional Description: All the functions you've studied in calculus so far live on a flat piece of paper. But you live in (at least!) three dimensions. Now you certainly know that calculus was invented to solve problems about the physical world, so we're going to have to move off that flat paper at some point. MATH 224 is where it happens. The key is the concept of a **vector**. If you've had a little bit of physics, you may have heard a vector is an object having direction and magnitude. You've also dealt with vectors in your study of linear algebra. In MATH 224, we'll tighten that definition up, and study functions whose domains and ranges consist of vectors. Can limit, derivative and integral make sense out here? The answer is yes, and when you're through you'll know how Newton's calculus – the greatest intellectual achievement of humankind! – made sense of Kepler's empirical observations about the motion of the planets – the greatest scientific discovery of all time! Come to think of it, maybe this course should be required for graduation...

Course Website: s26.middlebury.edu/MATH0224A

Instructor: Michael Olinick, Office: 202 Warner, Phone: 443-5559. Home telephone: 388-4290; email: molinick@middlebury.edu. My usual office hours will be Monday, Wednesday and Friday from 8:30 to 9:30 and 10:40 to 11 AM, and 12:05 to 1 PM on Monday and Wednesday. I would be happy to make an appointment to see you at other mutually convenient times.

Meeting Times: MATH 224A: MWF 9:45 – 10:35 AM (Warner 011)

Prerequisites: Calculus II (MATH 122) **and** Linear Algebra (MATH 200) or permission.

Textbook: Michael Olinick, *Multivariable Calculus: A Linear Algebra Based Approach*, Revised First Edition: Kendall-Hunt, 2023; ISBN 9781792437915 (<https://he.kendallhunt.com/product/multivariable-calculus-linear-algebra-based-approach>).

Warning: there are two printings of this edition; the newer one corrects some errors in the older one and includes additional problems. It's difficult, if not impossible, to tell the two printings apart by the cover or title page. The easiest way to identify the new printing is to examine the *dedication* page. If it includes **Alma** as a dedicatee, then you have the newer printing which is the one you want to have. Your daily assignments will include a few pages of reading in the text. Be certain to read the book carefully (with pencil and paper close by!) and to complete the relevant reading **before** coming to class and before embarking on the homework problems.

Supplemental Book: Jeffery Cooper, *A MATLAB Companion for Multivariable Mathematics* (Academic Press, 2001). We will distribute portions in class.

Computer Algebra Systems: Mathematically oriented software such as *MATLAB*, *Maple*, and *Mathematica* give you an opportunity to investigate the ideas of vector calculus in ways not available to previous generations of students. Relatively simple commands can direct a computer to carry out complex calculations rapidly and without error. More importantly, you can create and carry out experiments to develop and test your own conjectures. The very powerful graphics capabilities of these applications provide you with strong tools to deepen your understanding of multivariable calculus through visualization of curves and surfaces. *MATLAB* is an especially powerful computational and visualization tool which is used extensively in scientific research and engineering applications. Intermediate and advanced required and elective courses in Middlebury's Applied Mathematics Track adopt *MATLAB* as the de facto programming tool. There will be a required introductory *MATLAB* workshop late in the first week of the term; you will be able to choose either a Thursday evening or Friday afternoon session.

Requirements: There will be three midterm examinations and a final examination in addition to required daily homework assignments and an extended independent team project. The midterm examinations will be given in the evening to eliminate time pressure. Tentative dates for these tests are:

Monday, March 2

Monday, April 6

Monday, May 4

Final Exam: The registrar's office has **tentatively** set the date and time of our final exam:
Friday, May 15, 9 AM – Noon

Course Grades: Each of the midterm exams will be worth approximately 20%, the final about 30%, projects and quizzes roughly 10%. I will make adjustments with later work counting more heavily if students show improvement over earlier results.

The mathematics department regards a C grade as an indication of satisfactory *understanding* of the course material, a B as good/very good understanding and an A as an excellent/superior grasp of the material. Typically, but not always, these tend to correlate with averages in the 70's, 80's and 90's, respectively. I do strive to issue course grades keeping in mind the hundreds of students I have had in calculus classes over the years.

Accommodations Students who have Letters of Accommodation in this class are encouraged to contact me as early in the semester as possible to ensure that such accommodations are implemented in a timely fashion. For those without Letters of Accommodation, assistance is available to eligible students through the Disability Resource Center (DRC). Please contact ADA Coordinators Jodi Litchfield and Peter Ploegman of the DRC at ada@middlebury.edu for more information. All discussions will remain confidential.

Homework: Mathematics is not a spectator sport! You must be a participant. The only effective way to learn mathematics is to do mathematics. In your case, this includes working out many multivariable calculus problems.

There will be daily written homework assignments which you will be expected to complete and submit. They will be corrected and assigned a numerical score, but I view these assignments primarily as **learning** rather than testing experiences. I will occasionally assign some challenging problems which everyone may not be able to solve. You should, however, make an honest attempt at every problem.

Each homework assignment will probably take you between 2 and 3 hours to complete; this includes the reading and problem solving. If you keep pace with the course by spending an hour or so each day on it, then you will be quite successful. If you wait until the end of the week and then try to spend one 6 hour block of time on the material, then experience shows you face disaster!

Our homework grader this term is Zane Malarkey (zmalarkey@middlebury.edu).

Help: Please see me immediately if you have any difficulties with this course. There are ample resources on campus for assistance, including the Center for Teaching, Learning and Research in the Davis Family Library and the Q-Center (“Armstrong Quantitative Center”) in BiHall. The College will be assigning tutors to work with students in our course; they will schedule drop-in help sessions several nights a week. Stay tuned for more details.

One of the essential characteristics of college life that distinguishes it from secondary school is the increased responsibility placed on *you* for your own education. **Most of what you will learn will not be told to you by a teacher inside a classroom.**

Even if our model of you were an empty vessel waiting passively to be filled with information and wisdom, there would not be time enough in our daily meetings to present and explain it all.

We see you, more appropriately, as an *active* learner ready to confront aggressively the often times subtle and difficult ideas our courses contain. You will need to listen and to read carefully, to master concepts by wrestling with numerous examples and problems, and to ask thoughtful questions.

As you progress through the undergraduate mathematics curriculum, emphasis changes from mastering techniques to solve problems to learning the theory that underlies the particular subject you are studying. *Vector Calculus* is a transitional course. You will do plenty of calculations, find many derivatives and deal with a full quota of integrals. You will also find more of your effort directed toward understanding definitions, statements of theorems and their proofs. You will even be expected to come up with some short proofs of your own.

One of my goals for you this term is to develop your skills in *reading* mathematical expositions. I will expect that you will have read (perhaps more than once!) in advance the sections of the text relevant to the topic we will be exploring in class that day. I will not normally present a lecture which substitutes for reading the text. I will more likely use time in class to give a broader overview or alternative proofs or interesting applications and extensions of the material or previews of the next section.

Policy on Generative AI: Please see our [Policy on AI Usage](#) Any use of generative AI tools will be treated as a violation of Middlebury’s Honor Code.

MATH 224: Spring 2026
Tentative Course Outline
(Times are approximate)

- I. *Review (on your own as needed)*
 - Single Variable Calculus
 - Vectors
 - Equations and Matrices
 - Vector Spaces and Linearity

- II. *Derivatives (2 weeks)*
 - Functions of One Variable
 - Several Independent Variables
 - Partial Derivatives
 - Parametrized Surfaces

- III. *Differentiability (1 week)*
 - Limits and Continuity
 - Real-Valued Functions
 - Directional Derivatives
 - Vector-Valued Functions

- IV. *Vector Differential Calculus (2+ weeks)*
 - Gradient Fields
 - The Chain Rule
 - Implicit Differentiation
 - Extreme Values
 - Curvilinear Coordinates

- V. *Multiple Integration (3 weeks)*
 - Iterated Integrals
 - Multiple Integrals
 - Integration Theorems
 - Change of Variable
 - Improper Integrals

- VI. *Integrals and Derivatives on Curves (1 week)*
 - Line Integrals
 - Weighted Curves and Surfaces of Revolution
 - Normal Vectors and Curvature
 - Flow Lines, Divergence, and Curl

- VII. *Vector Field Theory (2+ weeks)*
 - Green's Theorem
 - Conservative Vector Fields
 - Surface Integrals
 - Gauss's Theorem
 - Stokes's Theorem