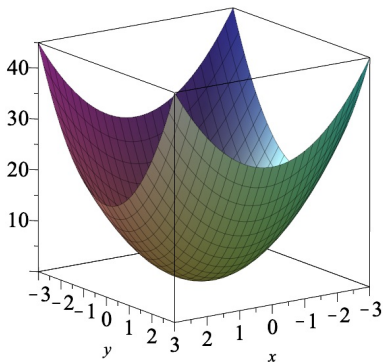


MATH 224: Vector Calculus



Class 5: February 18, 2026



Notes on Assignment 3
Assignment 4
Simple Animation in MATLAB

MATLAB Workshop Tonight from 7:00 to 8:30 [HERE](#)

Simple Animation in MATLAB

Animate Ellipse ($3 \cos t, 5 \sin t$), $0 \leq t \leq 2\pi$

```
axis([-6 6 -6 6])  
hold on  
N = 20;  
for i = 1:N  
    t = linspace(0, i*2*pi/N);  
    pause(0.2)  
    plot(3*cos(t), 5*sin(t), 'b', 'LineWidth', 3)  
end
```

Animate Strange Curve ($\cos 3t, \sin 5t$), $0 \leq t \leq 2\pi$

```
axis([-2 2 -2 2])  
hold on  
N = 20;  
for i = 1:N  
    t = linspace(0, i*2*pi/N);  
    pause(0.2)  
    plot(cos(3*t), sin(5*t), 'r', 'LineWidth', 3)  
end
```

Animate Helix ($\cos t, \sin t, t$) for $0 \leq t \leq 6\pi$

```
N = 30;  
for i = 1:N  
    t = linspace(0, i*6*pi/N);  
    pause(0.3)  
    plot3(cos(t), sin(t), t, 'LineWidth', 3)  
    xlim([-1 1]); ylim([-1 1]); zlim([0 20]);  
end  
xlabel('x');  
ylabel('y');  
zlabel('z');
```

Major Topic: Real-Valued Functions of Vectors

$$f : \mathbb{R}^n \rightarrow \mathbb{R}^1$$

(I) Defining and Using Derivatives

Chapters 3 – 5

(II) Integrals

Chapters 6 – 7

Initial Focus: Geometry of Such Functions

Some Useful Pictures

$$\mathbf{(A)} \quad f : \mathbb{R}^2 \rightarrow \mathbb{R}^1 \quad z = f(x, y)$$

Image: Interval of Real Numbers

Domain: Region in Plane

Graph: Surface in \mathbb{R}^3

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In General, graph of $f : \mathbb{R}^n \rightarrow \mathbb{R}^1$ is the set of points of the form $(\mathbf{x}, f(\mathbf{x}))$, an n -dimensional surface in $n + 1$ -dimensional space.

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(B) Contours

Level Curves for $f : \mathbb{R}^2 \rightarrow \mathbb{R}^1$

Level Surfaces for $f : \mathbb{R}^3 \rightarrow \mathbb{R}^1$

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Level Curves for $f : \mathbb{R}^2 \rightarrow \mathbb{R}^1$

Level Surfaces for $f : \mathbb{R}^3 \rightarrow \mathbb{R}^1$

C) Cross-Sections

Fix one variable

Example: $f(x, y) = 2x^2 + 3y^2$ or $z = 2x^2 + 3y^2$.

Observations:

1. $z \geq 0$ for all x, y ; $z = 0$ only at $(0, 0)$

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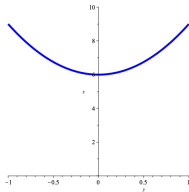
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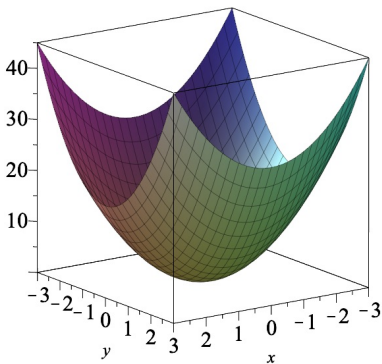
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3. Hold x fixed: $x = x_0$
 $2x_0^2 + 3y^2 = z$
Parabola in (y, z) -plane



4. Hold y fixed: $y = y_0$
 $2x^2 + 3y_0^2 = z$ Parabola in (x, z) -plane

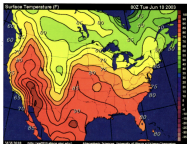


Level Curves

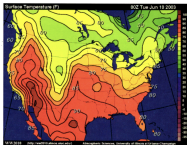
Given: output k (some constant)

Find: All Inputs Which Produce That Output

Examples Isotherms



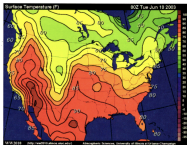
Examples Isotherms



Isobars

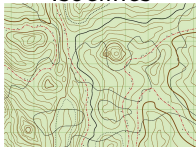
Examples

Isotherms

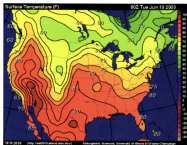


Isobars

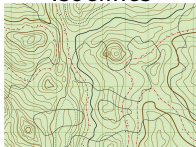
Isoclines



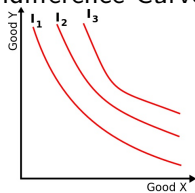
Examples Isotherms



Isobars Isoclines



Indifference Curves



**FUNDAMENTAL DIFFERENCE BETWEEN DOMAIN
BEING SUBSET OF \mathbb{R}^1 AND SUBSET OF $\mathbb{R}^n, n > 1$:**

Implications for Continuity, Derivative, Integral

These all depend on **LIMIT** :

$$\lim_{x \rightarrow a} f(x)$$

\mathbb{R}^1

$$\lim_{\mathbf{x} \rightarrow \mathbf{a}} f(\mathbf{x})$$

\mathbb{R}^n

2 ways to approach Infinitely Many Ways

Example:

$$f(x, y) = \begin{cases} \frac{xy}{x^2+2y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Approach along line $y = mx$:

$$f(x, mx) = \frac{x(mx)}{x^2 + 2(mx)^2} = \frac{mx^2}{x^2 + 2m^2x^2} = \frac{mx^2}{x^2(1 + 2m^2)} = \frac{m}{1 + 2m^2}$$

$$\frac{m}{1 + 2m^2} = \frac{1}{3}$$

$$2 \quad \frac{2}{1+8} = \frac{2}{9}$$

$$-1 \quad \frac{-1}{1+2} = -\frac{1}{3}$$

What are the possible values of $g(m) = \frac{m}{1+2m^2}$?

$$g'(m) = \frac{1-2m^2}{(1+2m^2)^2} \text{ and } g''(m) = \frac{4m(2m^2-3)}{(1+2m^2)^3}$$

- ▶ $g(m) \rightarrow 0$ as $m \rightarrow \pm\infty$
- ▶ $g'(m) > 0$ for $-\sqrt{2}/2 \leq m \leq \sqrt{2}/2$
- ▶ Point of Inflection at $\left(-\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}\right)$

