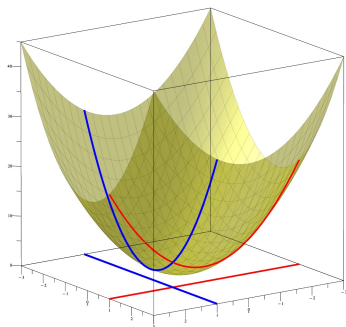


MATH 224: Vector Calculus



$$z = 2x^2 + 3y^2$$

Class 6: February 20, 2026



Notes on Assignment 5
Assignment 6
Some Surface Plotting Tools in MATLAB



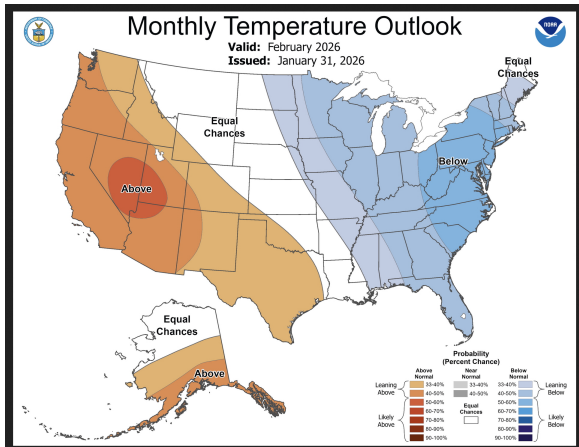
Exam 1
Monday, March 2
7 PM –

Level Curves

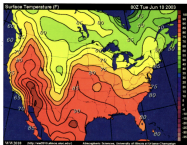
Given: output k (some constant)

Find: All inputs Which Produce That Output

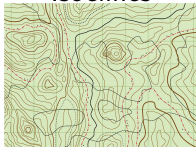
Examples Isotherms, Isobars, Isoclines, Indifference Curves



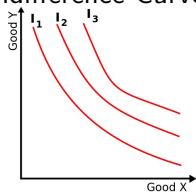
Examples Isotherms



Isobars Isoclines



Indifference Curves



**FUNDAMENTAL DIFFERENCE BETWEEN DOMAIN
BEING SUBSET OF \mathbb{R}^1 AND SUBSET OF $\mathbb{R}^n, n > 1$:**
Implications for Continuity, Derivative, Integral

These all depend on **LIMIT**:

$$\lim_{x \rightarrow a} f(x)$$

\mathbb{R}^1

$$\lim_{\mathbf{x} \rightarrow \mathbf{a}} f(\mathbf{x})$$

\mathbb{R}^n

2 ways to approach Infinitely Many Ways

Example:

$$f(x, y) = \begin{cases} \frac{xy}{x^2+2y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Approach along line $y = mx$:

$$f(x, mx) = \frac{x(mx)}{x^2 + 2(mx)^2} = \frac{mx^2}{x^2 + 2m^2x^2} = \frac{mx^2}{x^2(1 + 2m^2)} = \frac{m}{1 + 2m^2}$$

$$\frac{m}{1 + 2m^2} = \frac{1}{3}$$

$$2 \quad \frac{2}{1+8} = \frac{2}{9}$$

$$-1 \quad \frac{-1}{1+2} = -\frac{1}{3}$$

Example:

$$f(x, y) = \begin{cases} \frac{xy}{x^2+2y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Approach along line $y = mx$:

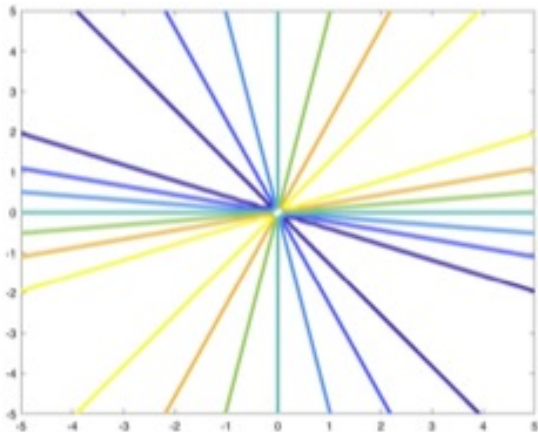
$$f(x, mx) = \frac{x(mx)}{x^2 + 2(mx)^2} = \frac{mx^2}{x^2 + 2m^2x^2} = \frac{mx^2}{x^2(1 + 2m^2)} = \frac{m}{1 + 2m^2}$$

m	$\frac{m}{1+2m^2}$
1	$\frac{1}{1+2} = \frac{1}{3}$
2	$\frac{2}{1+8} = \frac{2}{9}$
-1	$\frac{-1}{1+2} = -\frac{1}{3}$

Contours

$$f(x, y) = \begin{cases} \frac{xy}{x^2+2y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

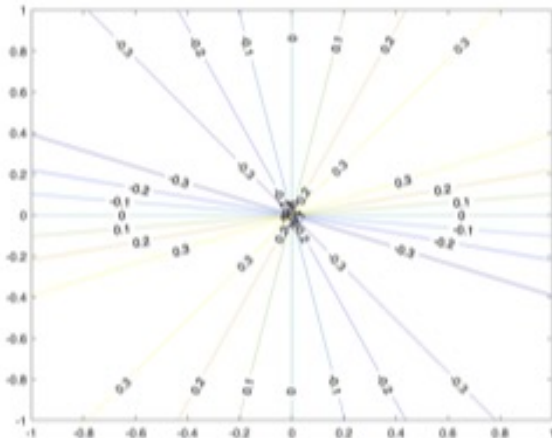
contour(x, y, f(x,y))



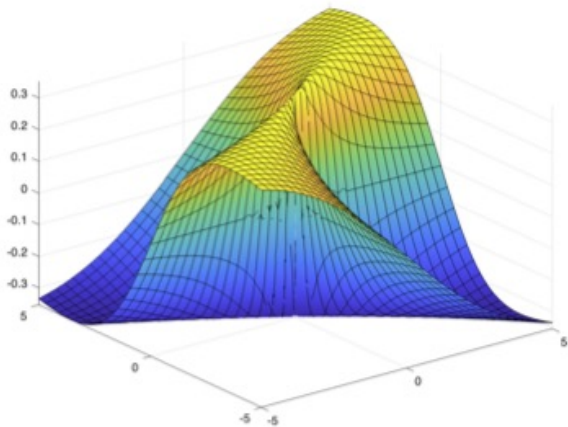
Contours

$$f(x,y) = \begin{cases} \frac{xy}{x^2+2y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

`contour(X,Y,f(X,Y), 'ShowText','on')`



$$f(x, y) = \begin{cases} \frac{xy}{x^2+2y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$



Derivative

$$f : \mathbb{R}^1 \rightarrow \mathbb{R}^1 : f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f : \mathbb{R}^1 \rightarrow \mathbb{R}^m : \mathbf{f}'(x) = \lim_{h \rightarrow 0} \frac{\mathbf{f}(x+h) - \mathbf{f}(x)}{h}$$

What about $f : \mathbb{R}^n \rightarrow \mathbb{R}^1$?

$$f'(x) \stackrel{?}{=} \lim_{\mathbf{h} \rightarrow 0} \frac{f(\mathbf{x} + \mathbf{h}) - f(\mathbf{x})}{\mathbf{h}}$$

Major Problems

Division by vector \mathbf{h} makes no sense.

Infinitely many ways $\mathbf{h} \rightarrow \mathbf{0}$.

Partial Solution

Consider 2 Special Ways for **h**

$$\mathbf{h} = (t, 0)$$

$$\lim_{t \rightarrow 0} \frac{f(x+t, y) - f(x, y)}{t}$$

$$\partial f / \partial x, f_x, D[1](f)$$

Partial Derivative

With Respect to x

Treat y as a constant

Use usual rules

of differentiation on x

$$\mathbf{h} = (0, t)$$

$$\lim_{t \rightarrow 0} \frac{f(x, y+t) - f(x, y)}{t}$$

$$\partial f / \partial y, f_y, D[2](f)$$

Partial Derivative

With Respect to y

Treat x as a constant

Use usual rules

of differentiation on y

Example: $f(x, y) = x^2y$ at point $(3, 4)$

$$\begin{array}{l|l} f_x(x, y) = 2xy & f_y(x, y) = x^2 \\ f_x(3, 4) = 2 \times 3 \times 4 = 24 & f_y(3, 4) = 3^2 = 9 \end{array}$$

$$\begin{aligned} f_x(x, y) &= \lim_{t \rightarrow 0} \frac{f(x+t, y) - f(x, y)}{t} \\ &= \lim_{t \rightarrow 0} \frac{(x+t)^2y - x^2y}{t} \\ &= \lim_{t \rightarrow 0} \frac{(x^2 + 2xt + t^2)y - x^2y}{t} \\ &= \lim_{t \rightarrow 0} \frac{x^2y + 2xyt + t^2y - x^2y}{t} \\ &= \lim_{t \rightarrow 0} \frac{2xyt + t^2y}{t} \\ &= \lim_{t \rightarrow 0} (2xy + ty) \\ &= 2xy \end{aligned}$$

Example: $f(x, y) = x^2y$

$$\begin{aligned}f_y(x, y) &= \lim_{t \rightarrow 0} \frac{f(x, y + t) - f(x, y)}{t} \\&= \lim_{t \rightarrow 0} \frac{x^2(y + t) - x^2y}{t} \\&= \lim_{t \rightarrow 0} \frac{x^2y + x^2t - x^2y}{t} \\&= \lim_{t \rightarrow 0} \frac{x^2t}{t} \\&= \lim_{t \rightarrow 0} x^2 \\&= x^2\end{aligned}$$

Example: $f(x, y) = x^2y$

$$f_x(x, y) = 2xy$$

$$f_y(x, y) = x^2$$

The **gradient** vector is

$$\nabla f = (f_x, f_y)$$

Let $\mathbf{w} = (x + 2t, y + 3t)$ so $\mathbf{h} = t(2, 3)$

$$\begin{aligned}f_{\mathbf{w}}(x, y) &= \lim_{t \rightarrow 0} \frac{f(x + 2t, y + 3t) - f(x, y)}{t} \\&= \lim_{t \rightarrow 0} \frac{(x + t)^2(y + 3t) - x^2y}{t} \\&= \lim_{t \rightarrow 0} \frac{(x^2 + 2xt + t^2)(y + 3t) - x^2y}{t} \\&= \lim_{t \rightarrow 0} \frac{12t^3 + 12t^2x + 4t^2y + 3tx^2 + 4txy + x^2y - x^2y}{t} \\&= \lim_{t \rightarrow 0} \frac{12t^3 + 12t^2x + 4t^2y + 3tx^2 + 4txy}{t} \\&= \lim_{t \rightarrow 0} (12t^2 + 12tx + 4ty + 3x^2 + 4xy) \\&= 4xy + 3x^2\end{aligned}$$

With $\mathbf{w} = (x + 2t, y + 3t) = (x, y) + t(2, 3)$,

$$f_{\mathbf{w}}(x, y) = 4xy + 3x^2 = (2xy, x^2) \cdot (2, 3) = \nabla f \cdot (2, 3)$$

COINCIDENCE?