

MATH 224

Some Hints and Solutions for Assignment 5

Chapter 2: Exercises 45, 51, Chapter 3: 4, 8 (new) = 6 (old) and Problem A

45: Follow the lead of the first example in the Projectile Motion section, to show you can write the position of the ball as $\mathbf{p}(t) = (x(t), y(t)) = (v \cos \frac{\pi}{6}t, v \sin \frac{\pi}{6}t - 16t^2 + 4) = (v \frac{\sqrt{3}}{2}t, \frac{v}{2}t - 16t^2 + 4)$. Now if we solve $x(t) = 310$, we can find the time t_0 at which the ball will reach the Green Monster. Show that $t_0 = \frac{620}{v\sqrt{3}}$. Then find v_0 so that $y(t_0) = 37.166$. You should get a value of about 118.6 feet per second.

51.: If d is the depth of the well, show that $t_1 = \frac{\sqrt{d}}{4}$ is the time it takes for the stone to reach the water. Then show that if t_2 is the time between the splash occurring and the noise being heard then $t_2 = \frac{d}{1125}$. Then solve the equation $t_1 + t_2 = 4$ for d .

Chapter 3: 4. Show that the determinant of M is 0 along the surface $z = xy/3$, Sketch a picture of the surface.

Chapter 3: 8 (newer printing) = 6 (older printing). The function $f(x) = \sqrt{9 - x^2 - y^2}$ is defined where ever $9 - x^2 - y^2 \geq 0$. The domain of f is a circular disk of radius 3 centered at the origin of the xy plane The graph of the function is a surface in \mathbb{R}^3 opening downwards over the domain (Cross sections of the graph are semicircles for fixed x and fixed y , circles for fixed z .) Because f is real-valued, the image of f is an interval of \mathbb{R} : $[0, 3]$.

Problem A: We'll use feet to measure distance and seconds to measure time.

Let θ be the angle at which the gun is fired.

Let v_0 be the initial speed of the shell.

Let h_{max} be the maximum height of the trajectory.

Let $\mathbf{x}(t)$ be the position vector of the shell.

Show $\mathbf{x}(t) = ((v_0 \cos \theta)t, -16t^2 + (v_0 \sin \theta)t)$ and $\mathbf{x}(186) = (186v_0 \cos \theta, -16(186)^2 + 186v_0 \sin \theta) = (396000, 0)$ yielding

$$v_0 \cos \theta = \frac{396000}{186}, 186v_0 \sin \theta = 16(186)^2$$

so $\tan \theta = \frac{2976}{2129} = 1.3978$ and hence $\theta \approx 54.4^\circ$

Explain why $h_{max} = 16(93)^2 + 2976(93) = 138385$ feet ≈ 26.2 miles