

## MATH 224

*Hints and Answers for Assignment 24*

## Change of Variable

## Exercise 21, 22, 24, 26 and 27 of Chapter 6

**Exercise 21:** Jacobian is  $\begin{pmatrix} \cos v & -u \sin v \\ \sin v & u \cos v \end{pmatrix}$  with determinant  $u$ .

**Exercise 22:** Jacobian is  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ v u^{v-1} & u^v \ln u \end{pmatrix}$ .

**Exercise 24:** Begin with a sketch of the region  $\mathcal{R}$ . Show that in polar coordinates, this region is described as

$$\{(r, \theta) : 0 \leq \theta \leq \pi/4, \sec \theta \leq r \leq 2 \sec \theta\}$$

and the function becomes  $\sec \theta$ . You may need to look up how to find an antiderivative of  $\sec^3 \theta$ .

**Exercise 26:** Show that the change of variable formula transforms  $I$  to

$$I = \int_{u=3}^{u=7} \int_{v=9}^{v=16} \frac{1}{2} dv du$$

which is half the area of the rectangle.

**Exercise 27:**

(a) Replace  $y$  with  $u/x$  in  $v = y^2 - x^2$  and obtain a quadratic in  $x^2$ .

(b) This part is a bit messy but straightforward. One expression for the determinant is

$$\frac{2u}{\sqrt{4u^2 + v^2} \sqrt{-2v + 2\sqrt{4u^2 + v^2}} \sqrt{2v + 2\sqrt{4u^2 + v^2}}}$$

. One chunk of the denominator has the form  $\sqrt{-A + B} \sqrt{A + B} = \sqrt{B^2 - A^2}$  where this last term reduces to  $\sqrt{16u^2} = 4u$ .

(d) Apply Change of Variable Theorem.