

MATH 224

Some Hints and Answers for Assignment 27

Exercises 9ac, 10df, 13 and 16 of Chapter 7.

9ac: Compute the indicated line integrals $\int_{\gamma} \mathbf{F}$ for the given vector fields and parametrizations \mathbf{g} for γ

(a) $\mathbf{F} = (x, 2y, 3z)$, $\mathbf{g}(t) = (t, t, t)$, $0 \leq t \leq 2$: $\int_0^2 (t, 2t, 3t) \cdot (1, 1, 1) dt = \int_0^2 6t dt = 12$.

(b) $\int_0^{\pi} -\sin t + \cos t dt = [\cos t + \sin t]_0^{\pi} = -2$.

10df: Sketch the curves γ_1 and γ_2 given respectively by the parametrizations $\mathbf{g}_1(t) = (\cos t, \sin t)$, $0 \leq t \leq \pi/2$ and $\mathbf{g}_2(s) = (1-s, s)$, $0 \leq s \leq 1$.

Hint: One curve is a quarter of a circle; the other is a line segment.

Then determine the line integral of the vector field \mathbf{F} along both γ_1 and γ_2 for each of the following vector fields:

(d) $\mathbf{F}(x, y) = (x, y)$: $\int_{\gamma_1} \mathbf{F} = 0$ and $\int_{\gamma_2} \mathbf{F} = 0$.

(f) $\mathbf{F}(x, y) = (xy^2, x^2y)$: Both line integrals are also 0.

11: Calculate the line integral of the gradient of the Cobb-Douglas function $f(K, L) = 2K^{2/3}L^{1/3}$ of Example 2 along

(a) the straight line between (1,1) and (27, 64)
Use the Fundamental Theorem, value is 70.

(b) the level curve where $f(K, L) = 1000$. Answer: 0

13: If \mathbf{F} is the vector field $F(S, I, R) = (-\beta SI, \beta SI - rI, rI)$, described in the SIR Epidemic Model, find the line integral of \mathbf{F} along the curve γ with parametrization $\mathbf{g}(t) = (t^2, t, t^2)$, $100 \leq t \leq 200$.

Note: You may wind up with the integral $\int_{100}^{200} -2\beta t^4 + \beta t^3 - rt + 2rt^2 dt$ which equals $-123625000000\beta + \frac{13955000}{3}r$.

16: Let \mathbf{F} be the vector field defined by $\mathbf{F}(x, y) = (-y^2, 2x^2)$.

(a) Sketch the vector field.
See the Assignment 27 sheet.

(b) Find two different curves g_1 and g_2 connecting the points (0,0) and (2,4) so that the line integrals of \mathbf{F} along the curves are different.

Hint: Consider straight lines and parabolas.

(c) Find a closed curve γ in the plane where the starting point is the same as the ending point but $\int_{\gamma} \mathbf{F}$ is not zero.

Solution: Let γ be the path that starts at (0,0), runs along g_1 to (2,4) and then runs back along g_2 to (0,0).

(d) Explain why parts (b) and (c) each allow you to conclude that \mathbf{F} is not a gradient field.

Hint: What must be true if it is a gradient field?