

MATH 224

Hints and Answers for Assignment 32

Exercises 27, 29a and 31 of Chapter 8.

27: If S is a sphere, what is its radius and where is its center?

$\sigma_s(s, t) \times \sigma_t(s, t) = (-a^2 \cos s \sin^2 t, -a^2 \sin s \sin^2 t, -a^2 \sin t \cos t)$ and $|\sigma_s(s, t) \times \sigma_t(s, t)| = a^2 \sin t$.
Thus $\text{Area}(S) = 4a^2\pi$.

$$\mathbf{29a:} \quad \iint_S f \, d\sigma = \int_{t=0}^{t=1} \int_{s=0}^{s=1} s + t\sqrt{4t^2 + 1} \, ds \, dt = \int_{t=0}^{t=1} \frac{1}{2} + t\sqrt{4t^2 + 1} \, dt = \frac{5}{12}(1 + \sqrt{5})$$

$$\mathbf{31:} \quad \int_S \mathbf{F} \cdot d\mathbf{S} = \int_{t=0}^{t=1} \int_{s=0}^{s=1} 12st - 2s^2 - t^2 \, ds \, dt = \int_{t=0}^{t=1} -\frac{2}{3} + 6t - t^2 \, dt = 2.$$