Minimizing Travel Costs

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The Problem:

We were tasked with finding the optimal location for a home base for the repair technicians regularly fixing 4 computer clusters in Detroit, MI, Middlebury, VT, Dallas, TX, and Tampa, FL. The technicians will make 3 trips to Vermont, 4 to Tampa, 2 to Dallas and 1 trip to Detroit every month. The technicians will be flying on a SyberJet SJ30i which uses $6.74 worth of fuel per mile traveled.

The Assumptions:

We made the assumption that the plane will travel in straight lines across an (x,y) coordinate system of the U.S. that we defined using the coordinates (W, N) for W (the longitudinal coordinate) and N (the latitudinal coordinate). We assume that every degree of latitude is 70 miles and every degree of longitude is 50 miles. We also made assumptions about the airports and their coordinates that the team would be flying into, and they are listed below:

Middlebury: Burlington International Airport (BTV) - 44.4720°N, 73.1533°W

Dallas: Dallas-Fort Worth International Airport (DFW) - 32.8972°N, 97.0377°W

Tampa: Tampa International Airport (TPA) - 27.9755°N, 82.5333°W

Detroit: Detroit Metro Airport (DTW) - 42.2124°N, 83.3534°W

$N =$ *Latitude of Home Base*

$W =$ *Longitude of Home Base*

$c =$ *Latitude of Cluster*

$d =$ *Longitude of Cluster*

$$D = \sqrt{(50d - 50W)^2 + (70c - 70N)^2}$$

$$D_N = \frac{-70(70c-70N)}{\sqrt{(50d-50W)^2+(70c-70N)^2}}$$

$$D_W = \frac{-50(50d-50W)}{\sqrt{(50d-50W)^2+(70c-70N)^2}}$$

$$K(W,N) = 13.48(D_{MI} + 2D_{TX} + 3D_{VT} + 4D_{FL})$$

$$K(W,N) = 13.48(3\sqrt{(50(-73.1533) - 50W)^2 + (70(44.4720) - 70N)^2} + 2\sqrt{(50(-97.0377) - 50W)^2 + (70(32.8972) - 70N)^2} + 4\sqrt{(50(-82.5333) - 50W)^2 + (70(27.9755) - 70N)^2} + \sqrt{(50(-83.3534) - 50W)^2 + (70(42.2124) - 70N)^2}$$

$$K_W(W,N) = 13.48(3\frac{-50(50(-73.1533)-50W)}{\sqrt{(50(-73.1533)-50W)^2+(70(44.4720)-70N)^2}} + 2\frac{-100(50(-97.0377)-50W)}{\sqrt{(50(-97.0377)-50W)^2+(70(32.8972)-70N)^2}} + 4\frac{-150(50(-82.5333)-50W)}{\sqrt{(50(-82.5333)-50W)^2+(70(27.9755)-70N)^2}} + \frac{-200(50(-83.3534)-50W)}{\sqrt{(50(-83.3534)-50W)^2+(70(42.2124)-70N)^2}})$$

$$K_N(W,N) = 13.48(3\frac{-70(70(44.4720)-70N)}{\sqrt{(50(-73.1533)-50W)^2+(70(44.4720)-70N)^2}} + 2\frac{-70(70(32.8972)-70N)}{\sqrt{(50(-73.1533)-50W)^2+(70(44.4720)-70N)^2}} + 4\frac{-70(70(27.9755)-70N)}{\sqrt{(50(-73.1533)-50W)^2+(70(44.4720)-70N)^2}} + \frac{-70(70(42.2124)-70N)}{\sqrt{(50(-73.1533)-50W)^2+(70(44.4720)-70N)^2}})$$

By setting $f_N = 0$ and $f_W = 0$, we found the coordinates for the home base to be 31.269965 °N, 83.17891 °W, resulting in Nashville, GA.

Elements of the Hessian Matrix for K(x,y) in Desmos:

$K_1(x,y) = \frac{d}{dx}\left(\frac{d}{dx}K(x,y)\right)$

$K_2(x,y) = \frac{d}{dx}\left(\frac{d}{dy}K(x,y)\right)$

$K_3(x,y) = \frac{d}{dy}\left(\frac{d}{dx}K(x,y)\right)$

$K_4(x,y) = \frac{d}{dy}\left(\frac{d}{dy}K(x,y)\right)$

$K_5(x,y) = K_1(x,y)K_4(x,y) - K_3(x,y)K_4(x,y)$

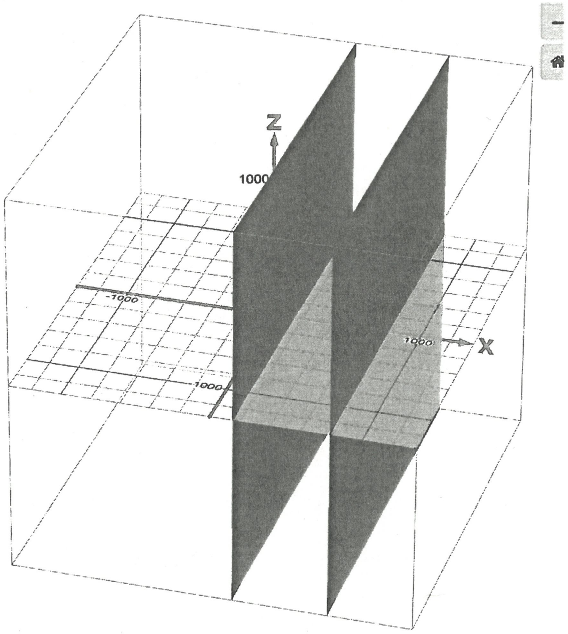
Eigenvalues of the Hessian Matrix:

$E(x,y,L) = \left(K_1(x,y) - L\right)\left(K_4(x,y) - L\right) - K_3(x,y)K_4(x,y)$

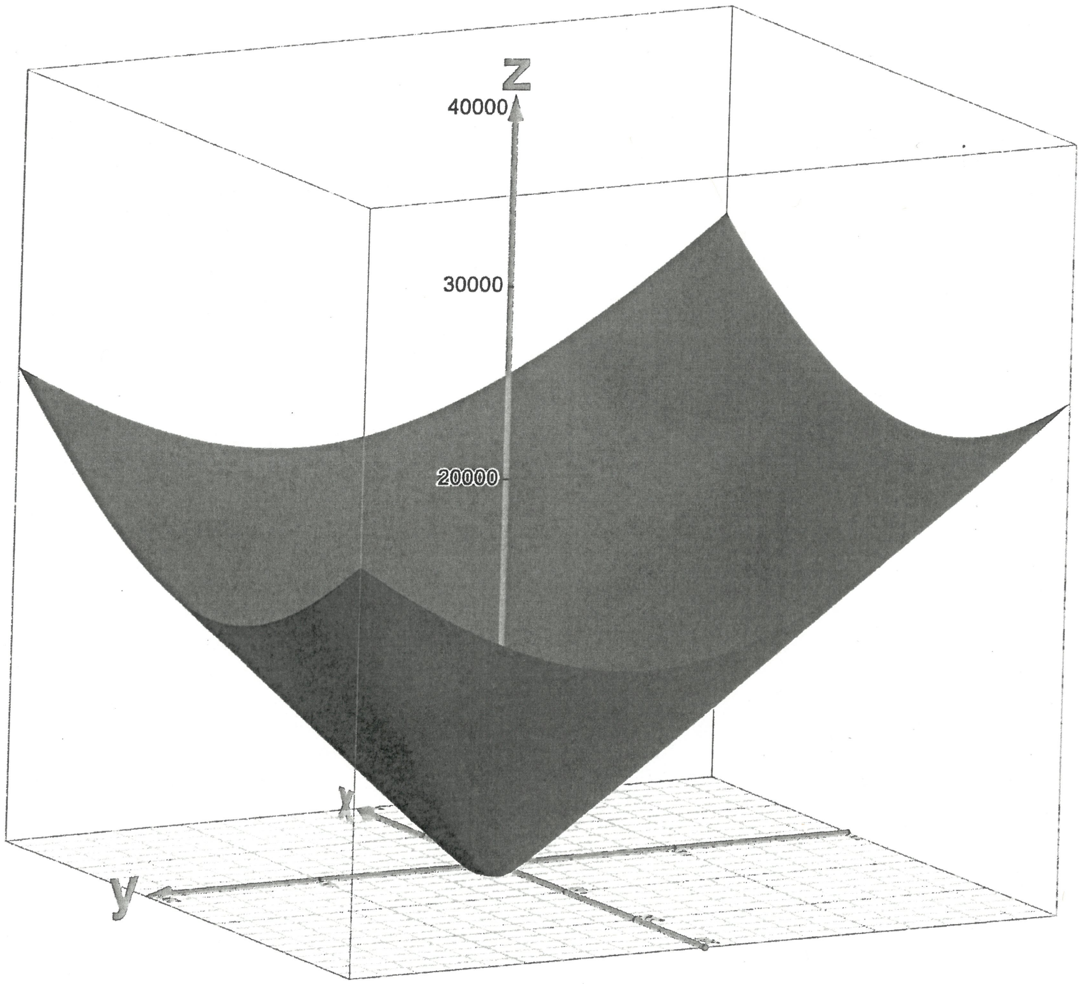
$0 = E(-83.35769, 35.72962, x)$

☑ Extend to 3D

By inspection of the graph of the eigenvalues, we can determine that all are positive, which means the Hessian matrix is positive definite and our calculated value is a minimum.



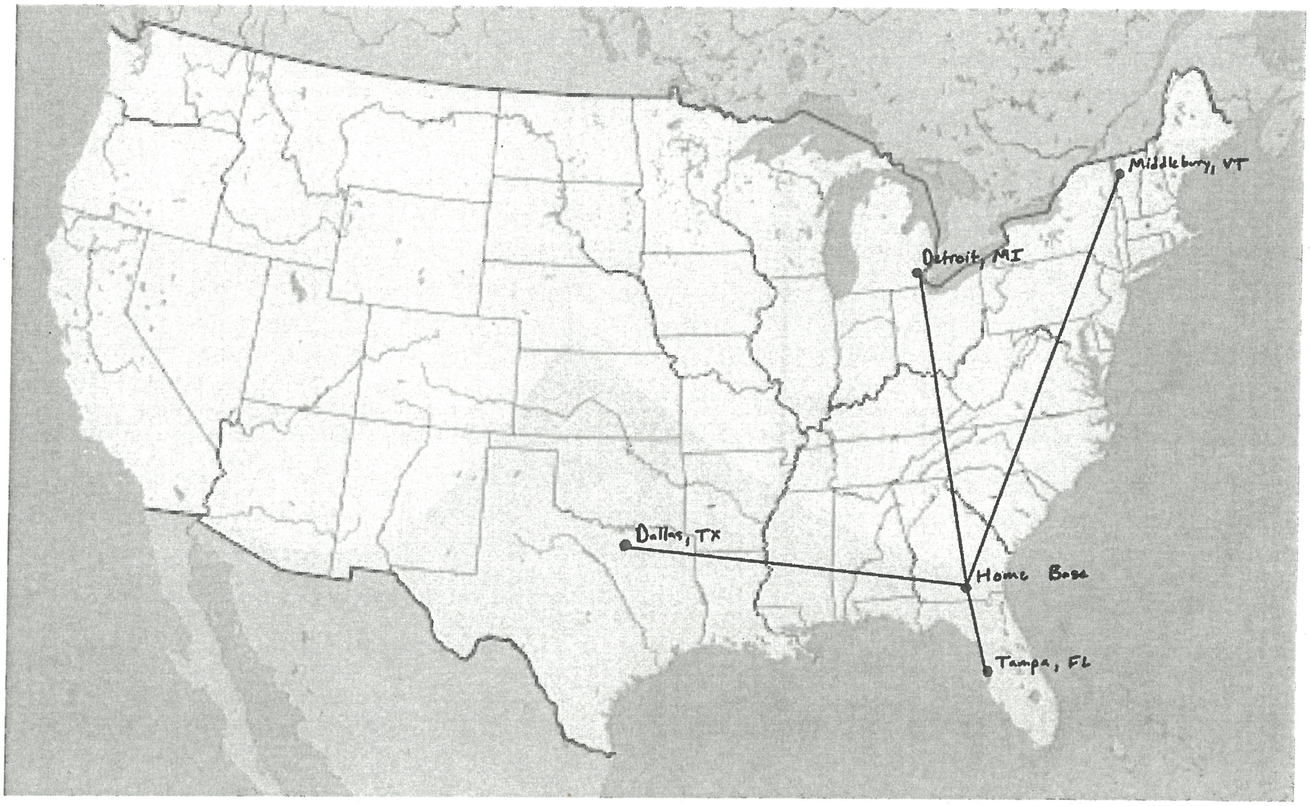
Minimum Monthly Cost Function:



(z values in graph above are scaled by 1/100 for imaging purposes)

Solution:

We determined the minimum final monthly travel cost for all locations to be $84,330.48, resulting in 31°16'11.9"N 83°10'44.1"W. The closest airport (Berrien County Airport, Nashville, GA) results in a total monthly travel cost of $84,331.91.



The following values represent the monthly travel cost if the headquarters were in Burlington (c_1, d_1), Dallas (c_2, d_2), Tampa (c_3, d_3), and Detroit (c_4, d_4).

$$K(c_1, d_1) = 113308.756908$$

$$K(c_2, d_2) = 114393.279139$$

$$K(c_3, d_3) = 85494.2142685$$

$$K(c_4, d_4) = 100856.6101$$

Links to Desmos Projects:

https://www.desmos.com/3d/udfunhtqqz

https://www.desmos.com/calculator/cigcdsgl3x