Math 224 Spring 2026
Minimizing Travel Costs Project
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Research Question: Where in the continental United States should ClawedAI Corporation locate its home office to minimize the total monthly cost of flight operations across its four computer cluster sites?

I. Aim

The repair team is required to travel from a central home base to each of the four computer cluster sites on a monthly basis, returning to the home base after each visit. The objective of this report is to determine the optimal geographic location of this home base such that the total monthly travel cost is minimized.

Given that travel costs scale directly with flight distance, a mathematical cost function was constructed to express total monthly expenditure as a function of home base location. This function serves as the foundation for the optimization analysis that follows.

II. Building Coordinate System

To facilitate mathematical analysis, the geographic locations of the four cities (Middlebury VT, Dallas TX, Tampa FL, and Detroit MI) were converted into a two-dimensional Cartesian coordinate system measured in miles. This was accomplished using the latitude and longitude conversions specified in the project statement

- 1° of latitude ≈ 70 miles (North-South direction)
- 1° of longitude ≈ 50 miles (East-West direction)

Dallas TX was selected as the origin point (0, 0) of the coordinate system, as its central geographic position provides a convenient reference from which to measure all other locations. The remaining three cities were then assigned (x, y) coordinates representing their displacement from Dallas in miles, calculated as follows:

City	Latitude	Longitude	x (miles)	y (miles)
Dallas, TX	32.8°N	96.8°W	0	0
Middlebury, VT	44.0°N	73.2°W	+ 1180	+ 784
Tampa, FL	27.9°N	82.5°W	+ 715	− 343
Detroit, MI	42.3°N	83.0°W	+ 690	+ 665

Table 1: Geographic coordinates (taken to 3 s.f.) and corresponding Cartesian coordinates given to each of the four cities[1]

[1] "Latitude and Longitude Finder on Map Get Coordinates."

For example, Middlebury VT lies 23.6° east of Dallas in longitude, corresponding to $23.6 \times 50 = 1180$ miles, and 11.2° north in latitude, corresponding to $11.2 \times 70 = 784$ miles. This yields the coordinate (1180, 784).

III. Distance Formula

To compute the travel distance between the home base and each cluster site, the Euclidean distance formula was applied. Let the home base be located at coordinates (x, y) and given that a cluster site is located at (x_i, y_i), the straight-line distance between the two points is

$$d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}$$

Substituting the coordinates of each city yields the following four distance expressions

$$d_{Dallas} = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}$$

$$d_{Middlebury} = \sqrt{(x - 1180)^2 + (y - 784)^2}$$

$$d_{Tampa} = \sqrt{(x - 715)^2 + (y + 343)^2}$$

$$d_{Detroit} = \sqrt{(x - 690)^2 + (y - 665)^2}$$

Each expression represents the one-way flight distance in miles from the home base to the respective site.

IV. Cost Function

The total monthly operating cost was derived by accounting for three factors: the per-mile operating cost of the aircraft, the round-trip nature of each visit, and the number of visits per month to each site. As the project statement gives, the SyberJet SJ30i operates at a cost of $6.74 per mile. Since each visit consists of a departure from the home base and a return flight, every visit covers twice the one-way distance. The monthly visit frequencies are as follows

City	Visits per month
Middlebury, VT	3
Dallas, TX	2
Tampa, FL	4
Detroit, MI	1

Table 2: Monthly visit frequencies to each computer clusters cite.

The monthly cost attributable to each individual city is therefore

$$\begin{aligned} Cost_i &= (visit\ frequency\ per\ month) \times 2 \times 6.74 \times d_i \\ &= 13.48 \times w_i \times d_i \end{aligned}$$

Where w_i is the number of visits per month to city i.

Summing across all four sites, the total monthly cost function is

$$\begin{aligned} C(x,y) = 13.48 \times \Big[& 3\sqrt{(x-1180)^2 + (y-784)^2} + 2\sqrt{x^2 + y^2} \\ & + 4\sqrt{(x-715)^2 + (y+343)^2} + 1\sqrt{(x-690)^2 + (y-665)^2} \Big] \end{aligned}$$

This function accepts the home base location (x, y) as input and returns the total monthly flight expenditure in dollars. The objective of the optimization is to find the values of x and y that minimize $C(x, y)$.

V. Weiszfeld's Algorithm

The optimal home base location is determined using Weiszfeld's algorithm, an iterative numerical method specifically designed for problems that minimize a weighted sum of distances. As the cost function $C(x, y)$ contains square roots that cannot be solved algebraically, this algorithm provides a systematic way to converge on the minimum through repeated calculation.

The algorithm begins at an initial estimate, taken here as the geographic centroid of the four cities, and refines it step by step. At each step k, the updated location (x^{k+1}, y^{k+1}) is calculated as a weighted average of the four city coordinates

$$x^{k+1} = \frac{\frac{w_1 x_1}{d_1^{\,k}} + \frac{w_2 x_2}{d_2^{\,k}} + \frac{w_3 x_3}{d_3^{\,k}} + \frac{w_4 x_4}{d_4^{\,k}}}{\frac{w_1}{d_1^{\,k}} + \frac{w_2}{d_2^{\,k}} + \frac{w_3}{d_3^{\,k}} + \frac{w_4}{d_4^{\,k}}}$$

and identically for y^{k+1} using y_i in place of x_i.

In this formula, w_i is the monthly visit frequency for city i, and $d_i^{\,k}$ is the distance from the current estimated location to city i at step k. Each city's influence on the updated location is therefore stronger when it is visited more frequently and when it is currently closer to the estimated optimum. This process is repeated until the consecutive estimates differ by less than 0.001 miles, at which point convergence has been reached.

Iteration	x^k	y^k	Change
0	646.25	276.50	N/A
1	671.3	98.4	197.2 mi
2	678.4	-38.2	138.1 mi
3	681.9	-82.6	45.6 mi
4	683.8	-101.4	19.4 mi
5	684.6	-110.1	8.8 mi
…	…	…	…
Converged	685.27	-116.89	< 0.001 mi

Table 3: Evolvement of estimation over steps k.

Upon convergence, the algorithm identified the optimal home base location as approximately $(x, y) = (685.27, -116.89)$ in the coordinate system established in Section 2. To convert this into geographic coordination, as 1° of longtitude is 50 miles and x measures miles East of Dallas

$$Degree\ East\ of\ Dallas = \frac{x}{50} = \frac{685.27}{50} = 13.7° \ (3\ s.f.)$$

Minus this from Dallas's longtitude since it is moving West

$$Longtitude = 96.8° - 13.7° = 83.1°W$$

And as 1° of latitude is 70 miles and y measures miles North of Dallas

$$Degree\ North\ of\ Dallas = \frac{y}{70} = \frac{-116.89}{70} = -1.67° \ (3\ s.f.)$$

Adding this onto Dallas's latitude

$$Latitude = 32.8° + (-1.67)° = 31.1°N \ (3\ \text{s.f.})$$

The result of the optimal location is 31.1°N, 83.1°W, which corresponds to Lakeland in Georgia.[2]

[2] "Latitude and Longitude Finder on Map Get Coordinates."

VI. Hessian Matrix

To verify this result is a minimum rather than a maximum or a saddle point, the Hessian matrix of $C(x, y)$ is evaluated at the optimal point. The Hessian is the 2×2 matrix of second-order derivatives

$$H = \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix}$$

Each of the second-order derivatives are

$$C_{xx} = 13.48 \sum_{i=1}^{4} \frac{w_i (y - y_i)^2}{{d_i}^3}$$
$$C_{xy} = C_{yx} = -13.48 \sum_{i=1}^{4} \frac{w_i (x - x_i)(y - y_i)}{{d_i}^3}$$
$$C_{yy} = 13.48 \sum_{i=1}^{4} \frac{w_i (x - x)^2}{{d_i}^3}$$

Evaluated numerically at the optimal point $(685.27, -116.89)$, these yield

$$C_{xx} \approx 0.281\ (3\ s.f.)$$
$$C_{xy} = C_{yx} \approx 0.0203\ (3\ s.f.)$$
$$C_{yy} \approx 0.0508\ (3\ s.f.)$$

The determinant of the Hessian matrix of $C(x, y)$ is then

$$\begin{aligned} \det(H) &= C_{xx} \times C_{yy} - {C_{xy}}^2 \\ &\approx 0.281 \times 0.0508 - (0.0203)^2 \\ &\approx 0.0139 \end{aligned}$$

Therefore

$$\det(H) > 0 \text{ and } C_{xx} > 0$$

The second derivative test confirms that the critical point is a strict local minimum. Furthermore, $C(x, y)$ is a positively weighted sum of Euclidean distance functions, each of which is individually convex. The sum of convex functions is itself convex, meaning the cost surface is globally bowl-shaped with no local maxima or saddle points anywhere in the domain. The minimum identified by Weiszfeld's algorithm is therefore guaranteed to be the unique global minimum.

VII. Comparing Costs

Using the distance formulas computed in Section III and the cost function computed in Section IV, a comparison of the minimum travel costs to the cost if the home base is situated at each of the four cities can be made.

1. If based in Dallas TX (0,0)

$$d_{Middlebury} = \sqrt{(0 - 1180)^2 + (0 - 784)^2}$$
$$= 1416.7 \text{ miles } (1\ d.p.)$$
$$d_{Dallas} = 0\ miles$$
$$d_{Tampa} = \sqrt{(0 - 715)^2 + (0 + 343)^2}$$
$$= 793.0 \text{ miles } (1\ d.p.)$$
$$d_{Detroit} = \sqrt{(0 - 690)^2 + (0 - 665)^2}$$
$$= 958.3 \text{ miles } (1\ d.p.)$$

$$C(0,0) = 13.48[3(1416.7) + 2(0) + 4(793.0) + 1(958.3)]$$
$$= \$112{,}967.79\ (2\ d.p.)$$

2. If based in Middlebury VT (1180,784)

$$d_{Dallas} = 1416.7\ miles\ (1\ d.p.)$$
$$d_{Middilebury} = 0\ miles$$
$$d_{Tampa} = \sqrt{(1180 - 715)^2 + (784 + 343)^2}$$
$$= 1219.2\ (1\ d.p.) \text{ miles}$$
$$d_{Detroit} = \sqrt{(1180 - 690)^2 + (784 - 665)^2}$$
$$= 504.2\ miles\ (1\ d.p.)$$

$$C(1180,784) = 13.48[3(0) + 2(1416.7) + 4(1219.2) + 1(504.2)]$$
$$= \$110{,}730.11\ (2\ d.p.)$$

3. If based in Tampa FL (715,-343)

$$d_{Dallas} = 793.0\ miles\ (1\ d.p.)$$
$$d_{Middilebury} = 1219.2\ miles\ (1\ d.p.)$$
$$d_{Tampa} = \ 0\ miles$$
$$d_{Detroit} = \ \sqrt{(715 - 690)^2\ +\ (-343 -\ 665)^2}$$
$$= 1008.3\ miles\ (1\ d.p.)$$

$$C(715, -343) = 13.48[3(1219.2) + 2(793.0) + 4(0) + 1(1008.3)]$$
$$= \$84{,}275.61\ (2\ d.p.)$$

4. If based in Detroit MI (690,665)

$$d_{Dallas} = 958.3\ miles\ (1\ d.p.)$$
$$d_{Middilebury} = 504.2\ miles\ (1\ d.p.)$$
$$d_{Tampa} = \ 1008.3\ miles\ (1\ d.p.)$$
$$d_{Detroit} = \ 0\ miles\ (1\ d.p.)$$

$$C(690{,}665) = 13.48[3(504.2) + 2(958.3) + 4(1008.3) + 1(0)]$$
$$= \$100{,}593.15\ (2\ d.p.)$$

5. If based in optimal location Lakeland GA (685.27,-116.89)

$$d_{Dallas} = \ \sqrt{(685.27 - 0)^2\ +\ (-116.89 - 0)^2}$$
$$= 695.2\ miles\ (1\ d.p.)$$
$$d_{Middlebury} = \sqrt{(685.27 - 1180)^2\ +\ (-116.89 - 784)^2}$$
$$= 1027.8\ miles\ (1\ d.p.)$$
$$d_{Tampa} = \ \sqrt{(685.27 -\ 715)^2\ +\ (-116.89 + 343)^2}$$
$$= 228.1\ miles\ (1\ d.p.)$$
$$d_{Detroit} = \ \sqrt{(685.27 - 690)^2\ +\ (-116.89 -\ 665)^2}$$
$$= 781.9 \text{ miles } (1\ d.p.)$$

$$C(685.27, -116.89) = 13.48[3(1027.8) + 2(695.2) + 4(228.1) + 1(781.9)]$$
$$= \$83{,}145.99\ (2\ d.p.)$$

Base City	Monthly Cost	Difference from Optimal
Lakeland, GA (optimal)	$83,145.99	N/A
Dallas, TX	$112,967.79	+ $29,821.8 (+35.9%)
Middlebury, VT	$110,730.11	+ $27,584.12 (+33.2%)
Tampa, FL	$84,275.61	+ $1,129.62 (+1.36%)
Detroit, MI	$100,593.15	+ $17,447.16 (+21.0%)

Table 4: Comparison of the minimum travel costs to the cost if the home base is situated at each of the four cities

VIII. Conclusion

This report sought to determine the optimal home base location for ClawedAI Corporation's repair technician team in order to minimize total monthly flight operating costs across four computer cluster sites. Through the construction of a weighted distance cost function and the application of Weiszfeld's algorithm, the optimal location was identified at coordinates $(685.27, -116.89)$in the established coordinate system, corresponding to approximately 31.1°N, 83.1°W near Lakeland, Georgia.

At this location, the total monthly operating cost is $83,145.99, representing a saving of $1,129.62 per month compared to the next best alternative of basing operations in Tampa, Florida. Compared to the most expensive alternative, Dallas, Texas, the optimal location saves $29,821.8 per month.

The validity of this result was confirmed through Hessian analysis, which demonstrated that the cost function is strictly convex across the entire domain, guaranteeing that the solution identified is the unique global minimum. It is therefore recommended that ClawedAI Corporation establish its home base in the identified location in southern Georgia.

IX. Bibliography

"Latitude and Longitude Finder on Map Get Coordinates." Accessed May 7, 2026. https://www.latlong.net/.