

# Minimizing Travel Costs

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The ClawedAI Corporation has asked us to help minimize their travel costs between their four sites in Middlebury Vermont, Dallas Texas, Tampa Florida, and Detroit Michigan. We will be advising them as to where to place their corporate office such that this cost is limited. They are visiting Middlebury three times, Dallas twice, Tampa four times, and Detroit once. To start, lets make some equations to help determine where our location should be:

```
%To start, we have to turn x and y into symbols so that we can do other
%functions with them later
syms x y
```

Using the distance formula, we can show the distance between each location and our hypothetical location at (x,y). To start, we need to change our coordinates by a factor of 50 for longitude and 70 for latitude:

```
%a:Middlebury: 73.168° W, 44.015°N (-3658, 3081);
%b:Dallas: 96.797° W, 32.777°N (-4840, 2294);
%c:Tampa: 82.458° W, 27.951° N (-4123, 1957);
%d:Detroit: 83.046° W, 42.331° N (-4152, 2963);

d1 = sqrt((x+3658)^2 + (y-3081)^2);
d2 = sqrt((x+4840)^2 + (y-2294)^2);
d3 = sqrt((x+4123)^2 + (y-1957)^2);
d4 = sqrt((x+4152)^2 + (y-2963)^2);
```

Now, we can combine these to make one equation for distance based on how often each location is visited. We can also show an equation for cost:

```
D = 3*d1 + 2*d2 + 4*d3 + d4;
C = 2*(6.74)*D;
```

Now that we have these equations, lets check each of the four cities to make sure that the minimum value is not at one of these locations by plugging the coordinates into our equations:

Middlebury as base:

$$D = 8.2135e+03$$

$$C = 1.1072e+05$$

Dallas as base:

$$D = 8.3887e+03$$

$$C = 1.1308e+05$$

Tampa as base:

$$D = 6.2401e+03$$

$$C = 8.4116e+04$$

Detroit as base:

$$D = 7.4686e+03$$

$$C = 1.0068e+05$$

It is now time to find our minimum from our function D. If we find the minimum from D, that will be our minimum for our cost, C. To do this, we will be using the Matlab function `fminsearch`. However, `fminsearch` uses different inputs. We need to convert our equation D into a numerical function so that `fminsearch` can read it, and we need to use that to make an optimizer function so that `fminsearch` is able to find the minimum values. To do this, we were able to use Matlab resources to teach us how to convert these functions. `fminsearch` also requires an initial guess to base the search off of, and we found this by averaging the points for the longitude and latitude.

```
Dfcn = matlabFunction(D);  
f = @(v) Dfcn(v(1),v(2));  
v0 = [-4200,2300];  
vmin = fminsearch(f,v0)
```

```
vmin = 1x2  
103 ×  
-4.1525 2.1778
```

Now that we have our location, we need to prove that this is a minimum. To do this, we will be using a Hessian matrix. We will plug our minimum location into the Hessian matrix (we used Matlab resources once again to teach us how to properly do this) and then taking the eigenvalues of the matrix. If the eigenvalues are both positive, then that means our matrix is positive definite and the curve slopes upwards in all directions, meaning we are in fact at a minimum.

```
H = hessian(D,[x,y]);  
Hnum = double(subs(H,{x,y},{-4153,2178}));  
eig(Hnum)
```

```
ans = 2x1  
0.0036  
0.0214
```

We found both our eigenvalues to both be positive, which confirms that we are at the minimum location. Since we found our minimum for D, we can now find our minimum for cost C. By plugging in our location into our equations for D and C shown above, we found the cost to be \$83,035 a month, which is less than any of the four locations given. Looking at our location on a map, we are located in Lakeland, Georgia. The closest airport to this location is in Valdosta, Georgia. Therefore, the corporate office should be located in Valdosta, Georgia.